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# An Essay of Geomechanics

*(Natural Philosophy)*



<http://www.glasnet.ru/~rwpbb/geo>

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1996

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*"...он руководствовался большей  
частою здравьи и смыслом,  
путеводителем и редко верным и  
почти всегда недостаточным..."  
А.С. Пушкин*

*"...He was led mostly by common sense—  
guidance seldom reliable and almost always  
insufficient..."  
A.S. Pushkin*

## Foreword

The profound division of labor in industry and science and the multiplicity of professional languages have restricted the communication between creative personalities and narrowed the scope of their interests; the fact that human life is stiffly regulated and that cultural demands have been degrading provides the milieu for unscrupulous manifestations of consumer psychology with respect to nature.

The outstanding achievements in certain fields of technology have resulted in the overrating of the role of specialized branches of knowledge, whose value is measured by their usefulness in obtaining ever more diversified material wealth. Society's interest in arriving at an integral perception of the universe has been declining. Theoretical generalizations have remained the domain of specialists. Primitive pragmatism on the one hand and mysticism on the other have been gaining ground.

Evidence of growing spiritual deficiency with advancing prosperity of our industrial civilization has been noted since long ago, but in the XX century humankind became aware of the looming global environmental crisis, the scale of which still remains underestimated. Responsibility for the situation that has arisen in the human habitat must be shared, among others, by social institutions in charge of public education.

## Introduction

### **1. Human habitat**

Human beings stand out in the natural environment because of their intelligence, which enables them to use experience accumulated by previous generations for predicting remote consequences of their activity, for reproduction of the environment to be inhabited by future generations, and for procreation of humankind.

Communication between contemporaries, assimilation of the experience of the living and past generations, and development of methods of generalization of experience and means of preservation of the knowledge indispensable for human life have all given rise to something immaterial, which can be designated as the spiritual sphere of the human habitat. At present, this sphere embraces religions, the arts, and science. The spiritual sphere has many dimensions and incessantly acquires new ones that are generated by the human genius.

Giving us unlimited capacities for creativity, the spiritual sphere at the same time develops its inner laws of harmony, preventing its decay and degradation. We contemplate the entirety of the spiritual sphere as the supreme Reason. Serving this constitutes the sense of human life.

At first, the spiritual sphere gravitated to those sources of empirical knowledge that nature endowed to human beings. Currently, natural science, despite its rapid development, plays too minor of a role in the spiritual sphere and does not properly influence the world-outlook prevailing in society.

### **2. Natural Science**

The spiritual sphere of the human habitat reflects the multi-faceted perception of the universe: art and religion make it possible for an individual to comprehend his or her unity with nature and all of humankind; natural science provides an individual with the generalized experience of communication with the physical world. The methodology of transforming empirical data into scientific knowledge is based on constructing models of physical bodies, phenomena, and processes of their interaction. Models are not copies of natural objects, but they reflect only their main properties in abstraction from a multitude of details. Models are used to study possible manifestations of these properties in variable conditions under the impact of diverse agents. This methodology proved to be rather fruitful, providing reliable knowledge beyond the scope of immediate experience.

Present-day models often actualize ideas and images that first emerged in the ancient world, providing evidence for the spiritual sphere to tend toward integrity and to preserve its structures.

Mechanics, which studies the simplest forms of motion, incessantly addresses the ideas of discreteness and continuity, volatility, fluidity and solidity, and gravity and zero gravity, which acquire their concrete sense and measure when applied to various bodies and phenomena.

Thus, the idea of atoms, indivisible particles that make up physical bodies, is presently interpreted in many ways, these particles being carriers of distinctive properties: nucleons define the parameters of atoms; atoms, of elements; elements, of chemical compounds; and cells, of living organisms.

Properties of physical bodies depend not only on atoms (in the broad sense) but also on their mutual arrangement and character of motion.

Mechanics uses broadly the models of ideal gas, ideal fluid, and ideal solid.

Let us consider these models in more detail and thus approach the discussion of models used in geomechanics and of the problem of reproduction of the human habitat.

## **Ideal Gas**

In the model of monatomic ideal gas, particles exchange kinetic energy only by colliding with each other. If particles occur in a closed heat-insulated volume, time-averaged velocities of all particles are equal and constant.

In colliding with walls that limit the volume, particles transmit impulse to the walls, a phenomenon perceived as pressure of the gas  $p$ .

To characterize the energy of chaotic motion of atoms in a unit of volume, temperature  $T$  is used, the value of which is proportional to the square of the average velocity of atoms. Energy exchange between the gas and immobile walls continues as long as their temperatures remain different: energy is transferred from the body with high temperature to the cold body. This empirical law is used in various instruments that measure temperature of gas and, consequently, the energy of motion of atoms in the gas.

To characterize properties of gas, in addition to pressure and temperature, density  $\rho$  is used, which represents the total mass of particles occurring in the unit of volume. The number of particles in a unit of gas volume at a fixed pressure and temperature is the same for all gases; hence, gas density is proportional to the mass of a single atom. Although distance between atoms in a gas at atmospheric pressure is about 10 times that in a solid, these distances are so small that, in everyday life, gas is perceived as a continuous elastic matter that fills freely the whole space available—a kind of ether, an image born by imagination in ancient times.

All the properties of a gas can be expressed through parameters of motion of atoms, which have a kinetic energy and momentum (impulse). The work done on, or by, a gas (compression or expansion) brings about a change not only in density but also in pressure and temperature. The interrelation of these parameters of gas is termed its equation of state:

$$P/\rho = nkT ,$$

where  $n$  is the number of particles in a unit of volume,  $P$  is pressure per unit of area,  $T$  is temperature, and  $k$  is Boltzmann's constant.

Macroscopic motion of compressible gas can be regarded as a mechanical flow of continuous medium, provided the relationship  $p = p(\mathbf{r})$ , i.e., the mechanism of energy exchange between gas particles, is known. In macroscopic motion of a real gas, the relationship  $p = p(\mathbf{r})$  can be violated for many reasons.

In polyatomic molecules, the number of degrees of freedom increases notably—the three translational degrees (as with atom) are supplemented with three rotational and some vibrational degrees. As vibrations are excited, the molecule can break into pieces, and new particles are thus formed. On the other hand, at low temperatures, some vibrational degrees of freedom remain "frozen" and are not involved in energy exchange. Fast atoms colliding can result in electron detachment and the formation of electrically charged particles. Lastly, heat radiation of a heated gas removes some energy from the moving gas. Seemingly, the above "imperfections" of real gases narrow strongly the field in which the ideal gas model is applicable. This is, however, not true. The successful application of the ideal gas model follows from its simplicity: all the properties of gas and its behavior are determined through kinetic energy, which is distributed equally between all the degrees of freedom of atoms, and the macroscopic motion as such does not affect the energy exchange between atoms. After the properties of ideal gas had been canonized and type processes had been modeled, it became possible to study and understand many "imperfections" of real gases by comparing them with ideal gas. Apparently, the principal advantage of the once chosen scientific approach to the physical world around us lies in constructing and studying models.

## ***Ideal Fluid***

The ideal fluid model embodies the idea of fluidity. The following properties of ideal fluid are realized in a broad class of macroscopic motions – in our case, flows: constant density, lack of viscosity, the spreading over horizontal plane by gravity, and strong bonds between fluid particles providing continuity of flowing fluid. Fluid particles possess only kinetic and potential energy, hence preservation of mechanical motion energy is predetermined.

In those cases when motion of fluid particles testifies to the absence of vortexes and the flow is defined by the shape of vessel or channel, the flow parameters are predictable.

The ideal-fluid model made it possible to study a multitude of complex hydrodynamic problems of crucial applied importance. At the same time, ideal fluid allows for the existence of vortex structures, which shed light on integral mechanical properties of continuous media under natural conditions. In ideal fluid, low-velocity motion of a well streamlined body experiences no resistance: the trajectory of fluid particles flowing around a spherical body is absolutely symmetrical, and hence pressures on the frontal and rear surfaces are equal. Lack of viscosity also precludes friction of fluid particles against the side surfaces.

At great velocities, a cavitation bubble appears behind the moving body in the fluid, where pressure is zero; hence, resistance to the motion of the body is defined by pressure on the frontal surface. Trajectories of moving particles at the boundary of the cavitation bubble converge some distance behind the body and are ejected as a jet after the body.

If at a certain instant the streamlined body is removed, the flow structure that emerged will continue to exist. The inner layers (adjoining the bubble) will form a flow that is closed on itself: the jet that arose through convergence of trajectories of moving particles, intruding the quiescent fluid, will act as a streamlined body. Then fluid particles of the jet will spread over the bubble's surface to converge again in the rear part of the moving bubble, in which, you will recall, there is no pressure.

On the outer boundary of the vortex flow, velocities equal those of the fluid flowing around this vortex. As a result of formation of the cavitation vortex, there emerges a structure that separates moving particles kinetically: there coexist two flows of distinct natures whose particles neither intermix nor exchange energy.

If this paradoxical situation had no chance to arise in a real, nonideal fluid, it would not present even theoretical interest. However, the above kinematics of motion of cavitation vortex requires practically none of the fundamental properties of ideal fluid, except incompressibility. Therefore, such structures are possible, which was recently proved experimentally.

The dimension of cavitation vortex changes as a function of pressure in the fluid that flows around it: volume decreases as pressure increases and increases as pressure decreases. The presence of such vortex structures in an ideal fluid imparts it properties of a compressible elastic medium.

On the free surface of an ideal fluid, in a gravity field, gravity waves appear behind any floating body. Amplitude of fluid-particle vibration in these waves decreases with increasing distance from the surface-the more rapid the decrease, the shorter the wavelength. Propagation velocity of the surface waves is proportional to the square root of the wavelength multiplied by gravitational acceleration.

Formation of these waves consumes the energy of the floating body; hence, there is resistance to the translation of this body. Damping of perturbed surface of an ideal fluid can be achieved through dissipation of gravitational waves over a limitless surface. There is no other mechanism for damping of vibration in an ideal fluid.

Any rigid body moving inside a vessel filled with a fluid causes each fluid particle to move in the vessel's volume as a consequence of incompressibility of the fluid. If this body follows a closed trajectory, after each cycle fluid particles, irrespective of their position, will shift to a new location, thus stirring the fluid throughout the vessel's volume. Macroscopic motion of individual volumes of incompressible fluid is connected rigidly with motion of all particles in the closed volume.

In compressible fluid, any change in mechanical motion at a certain point in space gives rise to a disturbance wave whose front propagates at the velocity of sound. The event in the perturbed region bears no influence on the fluid particles outside this region. Hence, in compressible fluid, a sort of spatial structuring of the fluid takes place. This is manifest most clearly in supersonic flows, where regions are separated by shock waves that are impenetrable to disturbances.

The recollection of characteristics of supersonic flows and perturbances that always occur in moving compressible fluid is helpful in determining the scope of applicability of the ideal-fluid model, which can be sometimes regarded as a

limiting transition of a real compressible fluid when the velocity of sound in it tends to infinity.

The structuring of an ideal fluid, even without eliminating such properties as incompressibility and lack of viscosity, leads to nonpreservation of mechanical energy in the fluid flow.

Lack of molecular viscosity predetermines the fact that vortex motion constantly arises everywhere in the fluid flow. It is natural to assume that a vortex structure evolves from major vortexes, which generate minor vortexes, which give rise to even smaller ones, etc. Rearrangement of mechanical energy of the flow proceeds in the same direction. As an ideal fluid has no limitation to determine the minimum size of the vortex, this structuring process can be continued to infinity. Steady-state vortex structure will take place provided the energy transmitted in a unit of time at each level is the same.

Let us designate the radius of  $i$ th-level vortex as  $r_i$ , and its kinetic energy as  $E_i$ . In one order of magnitude,  $E_i \approx \rho v_i^2 r_i^3$ , where  $v_i$  is the velocity of fluid particles in the vortex in the environment of macroscopic transfer of the vortex structure. Characteristic time of energy transmission to the vortex is defined by the synodic period of fluid particles in the vortex,  $2\pi r_i/v_i$ .

Suppose that vortexes of each size occupy the same fraction of the fluid volume. Then the number of vortexes of the same size will be proportional to  $1/r_i^3$ .

The energy flow transmitted from vortexes of one level to smaller vortexes of another level must be proportional to the energy of vortex and the number of vortexes and inversely proportional to the time of energy transmission:

$$\rho v_i^2 r_i^3 \frac{v_i}{2\pi r_i} \frac{1}{r_i^3} = \frac{\rho v_i^3}{2\pi r_i} = const$$

$$v_i^3 \sim r_i .$$

It follows that, in a steady-state mechanical-energy flow passing through a vortex structure, total energy of vortexes of the same size will decrease with  $r_i$ :

$$\rho v_i^2 r_i^3 (1/r_i^3) = \rho v_i^2 \sim \rho r_i^{2/3} .$$

Total energy of the whole vortex structure in the context of assumptions just made will be equal, in the order of magnitude, to kinetic energy of the flow that gave rise to this structure. Despite the fact that an ideal fluid lacks viscosity, vortex structure, by consuming energy and transferring it to ever smaller vortexes, damps the motion of large-scale vortexes and decelerates the flow that produced the vortexes. Generally, an ideal fluid with vortex structure can fairly well model, at the macrolevel, the motion of viscous fluid.

Vortex motion can perturb the continuity of fluid. Although velocity of fluid particles decreases with dimension of the vortex ( $v_i \sim r_i^{1/3}$ ), centrifugal forces  $\rho(v_i^2/r_i) \sim 1/r_i^{1/3}$  increase and can exceed not only pressure in the fluid but also cohesion forces. Unlike ideal fluid, water, for example, under natural conditions has a tensile strength measured in  $\text{kg/cm}^2$ .

## Ideal Solid

Undeformable solid has three translational and three rotational degrees of freedom (for a body with axial symmetry). Macroscopic motion of a solid differs notably from the motion of a point of mass. Suffice it to compare the motion of a point of mass on an inclined plane with that of a ball and a spherical shell of the same size and weight in order to comprehend the significance of spatial distribution of mass in a solid body. If the body has a more complex shape, energy can be distributed unequally between degrees of freedom, and hence the body's motion in the gravity field can change with time.

Deformable solid changes its shape under the action of external forces. The ability to resist deformation is a function of inner bonds between portions of the body.

When deformation of a solid body is small, it develops internal stresses counterbalancing the action of external forces, which disappear as soon as external forces cease to act. In case a body resumes its initial shape upon disappearance of internal stresses, this body is perfectly elastic.

Formulated by Hooke, the law of linear proportionality of elastic stresses and strains lays the foundation for the ideal elastic-body model.

Take an isotropic elastic cube with origin of coordinates placed at one of the eight apices and coordinate axes directed along the cube's axes and designated  $x$ ,  $y$ ,  $z$ . Deformation of such a cube will be characterized by the following parameters: elongation  $u_i$  of the body along the axes, which assumes the form  $u_x$ ,  $u_y$ ,  $u_z$  over the three axes, designates displacement of points inside the cube along one of the axes, as the origin of coordinates remains fixed.

Relative elongation of the body along the  $x$  axis is:

$$u_{xx} = \frac{\partial u_x}{\partial x}.$$

That this is written as a derivative implies that there exists elongation of a cube as small as is wished. Likewise, elongation of the cube over the two other axes can be characterized as follows:

$$u_{yy} = \frac{\partial u_y}{\partial y}; \quad u_{zz} = \frac{\partial u_z}{\partial z}.$$

Needless to say, with a minus sign in place of the plus sign, this deformation will signify shortening.

Distortion of the cubic shape is characterized by shear deformation; points located on a straight line that parallels one of the axes (e.g.,  $x$ ) become increasingly displaced from the origin of coordinates toward the  $y$  axis.

$$u_{xy} = u_{yx} = \frac{1}{2} \left( \frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x} \right)$$

(Usually, a symmetrical form is used).

The remaining shear deformations are written in the same manner:

$$u_{xz} = u_{zx}; \quad u_{yz} = u_{zy}.$$

Internal stresses in the cube are characterized by forces that must be applied to the faces of the cube in order to obtain the strain measured. On any flat surface,

three forces can be specified: one acting normally to the surface, and two at right angles to one another lying in the plane. Related to a unit of area, these forces are termed stresses.

In the cube under consideration, for the face normal to the  $x$  axis, these three stresses can be written as  $\sigma_{xx}$ ,  $\sigma_{xy}$ ,  $\sigma_{xz}$ .

Linear relationship between stresses and strains in the Hookean elastic body has the following form:

$$\sigma_{xx} = \frac{E}{(1+\sigma)(1-2\sigma)} [(1 - \sigma)u_{xx} + \sigma(u_{yy} + u_{zz})]$$

$$\sigma_{yy} = \frac{E}{(1+\sigma)(1-2\sigma)} [(1 - \sigma)u_{yy} + \sigma(u_{zz} + u_{xx})]$$

$$\sigma_{zz} = \frac{E}{(1+\sigma)(1-2\sigma)} [(1 - \sigma)u_{zz} + \sigma(u_{xx} + u_{yy})]$$

$$\sigma_{xy} = \frac{E}{1+\sigma} u_{xy}; \quad \sigma_{yz} = \frac{E}{1+\sigma} u_{yz}; \quad \sigma_{zx} = \frac{E}{1+\sigma} u_{zx}.$$

Let us give formulas for strains expressed through stresses, as obtained by algebraic transformation of the above formulas:

$$u_{xx} = \frac{1}{E} [\sigma_{xx} - \sigma(\sigma_{yy} + \sigma_{zz})],$$

$$u_{yy} = \frac{1}{E} [\sigma_{yy} - \sigma(\sigma_{zz} + \sigma_{xx})],$$

$$u_{zz} = \frac{1}{E} [\sigma_{zz} - \sigma(\sigma_{xx} + \sigma_{yy})].$$

All of the formulas contain experimentally determined constants characterizing elastic properties of material: Young's modulus  $E$  and Poisson's coefficient  $\sigma$ .

Young's modulus, as follows from the above formulas, is determined by extension or compression of the bar, when there are no stresses on the side surface. Under extension along the  $x$  axis,

$$u_{xx} = \frac{1}{E} \sigma_{xx} \quad (\sigma_{yy} = \sigma_{zz} = 0).$$

The same experiments are used to determine the Poisson coefficient; when side stresses are absent,

$$\sigma_{yy} = \frac{E}{(1+\sigma)(1-2\sigma)} [(1 - \sigma)u_{yy} + \sigma(u_{zz} + u_{xx})] = 0.$$

$$u_{yy} = u_{zz}$$

Hence, it follows that  $u_{yy} = u_{zz} = -\sigma u_{xx}$ . The bar under tension decreases in cross-section so that the ratio of the strains equals  $\sigma$ . For isotropic medium, these two parameters completely define elastic properties. Physically more clear are the volume compression modulus  $K$  and shear modulus  $\mu$ , which are expressed through  $E$  and  $\sigma$  as follows:

Fig. 1.

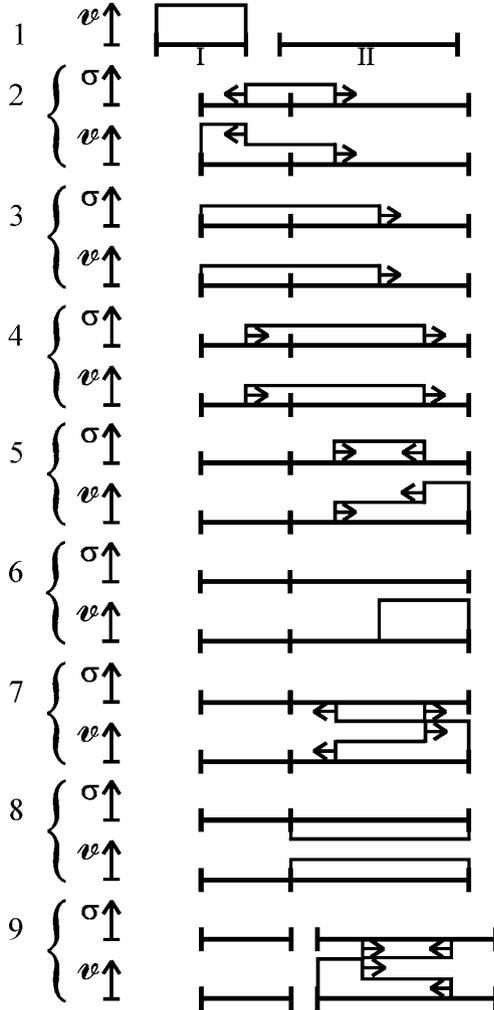
**Collision of bars I and II**

Bar I moves at a velocity of  $2v$  and collides with bar II.

In both bars, elastic waves arise, which decelerate bar I and accelerate bar II.

Wave motion at consecutive instants of time is represented by epures of mass velocities  $v$  and of stresses  $S$ .

Motion direction of the wave front is indicated with arrows.



$$K = \frac{E}{3(1-2\sigma)}; \quad \mu = \frac{E}{2(1+\sigma)}.$$

Note that Poisson's coefficient cannot exceed 0.5, because at this value the denominator of the formula for volume modulus becomes zero. Hence, Young's modulus and shear modulus also must become zero. Solid ceases to be solid.

The density of material and elastic properties determine the velocity of wave propagation in a solid. In limitless medium, two kinds of waves are possible: longitudinal, in which particles are displaced in the direction of wave propagation, and transverse, in which particles are displaced normally to the direction of wave propagation.

$$\text{Longitudinal wave velocity } c_l = \sqrt{\frac{E(1-\sigma)}{\rho(1+\sigma)(1-2\sigma)}}.$$

$$\text{Transverse wave velocity } c_t = \sqrt{\frac{E}{2\rho(1+\sigma)}}.$$

In bodies of finite dimensions and different shapes, there can arise specific waves, whose velocity of propagation can differ notably from velocities in limitless medium: surface waves of various types, flexural waves in plates and bars, torsional waves, etc. At the boundaries of bodies, waves are reflected and refracted, causing the wave pulse, initially simplest in form, reflecting incessantly off the body's boundaries, to excite in the body a broad spectrum of vibrations, tending to distribute its energy among a great number of degrees of freedom of the solid. Account must be made of the fact that high-frequency vibrations can be dissipated by the grained structure of polycrystalline bodies, by various structural inhomogeneities, and by crystal lattice defects. In this respect, energy capacity of solid is very large.

In ideal solid, absorption of mechanical energy is impossible, and only an energy dissipation takes place. Vibrations of a solid and its motions as a whole define all its possible states.

If a body is free and moves by inertia through space, kinetic energy of its energy as a whole is preserved, so that vibrational motion of the body can be ignored. However, when two such bodies interact, some portion of the translational-motion energy can be transmitted to inner degrees of freedom, so that doubt arises as to the ideal elasticity of the bodies. Figure 1 illustrates this situation, presenting a wave mechanism of energy transmission, through collision, from a short bar that had a velocity of  $2v$  to a bar of twice the length. Whereas at the initial instant the entire energy was the kinetic energy of bar I, at the instant of their parting this energy is found equally distributed between the kinetic energy of translational motion and the elastic energy of the extended bar II. Judging from the velocity of motion of the center of mass of the bar, the effect of collision is a "loss" of 50% of the initial energy.

The vibrational process that sets on in bar II will tend to dissipate the initially simple wave impulse and excite a broad spectrum of vibrations; in an ideal solid devoid of the mechanism that absorbs mechanical energy, these vibrations will form, with time, a steady-state dynamic pattern. In regularly shaped bodies, this spectrum will display natural frequencies of the body and overtones.

Energy transmission from low- to high-frequency vibrations in ideal solids is not limited by any factor. Therefore, the hierarchical structure of vibrations is capable of "absorbing" a large amount of energy, and, in the case of colliding bars just discussed, vibrational process in bar II can be damped in this manner.

Quite intriguing is the very functioning of a hierarchical dynamic structure in steady-state or quasisteady-state mode, when the mechanical-energy flow distributed by this structure feeds from a major source whose depletion over the life time of the structure is practically imperceptible.

### **Dust Cloud in Proper Gravity Field**

Recall that, in agreement with Newton's law of gravitation, two masses are attracted together with a force

$$F = \frac{GMm}{r^2}.$$

Here,  $G = 6.67 \cdot 10^{-8} \text{ erg} \cdot \text{cm}/\text{g}^2$ ;  $m$  and  $M$  are measured in grams;  $r$ , in centimeters; and  $F$ , in dynes. The force of interaction is directed along the line connecting the centers of mass.

Gravity field is characterized by a potential

$$U = -\frac{GMm}{r} \quad \left( F = \frac{\partial U}{\partial r} \right).$$

In case  $M \gg m$ , motion of the interacting bodies is simplified: the massive body can be deemed immobile, and the entire kinetic energy resides in the light body. The latter's motion depends on the value of angular momentum

$$I = mr^2 \dot{\phi} \quad (\dot{\phi} \text{ is angular velocity}).$$

Gravitational forces acting between spherical undeformable bodies cannot change the angular momentum; hence, as the distance  $r$  changes, some potential energy becomes "bonded" by the condition of constancy of  $I$ . Subtracting this energy, we obtain the effective potential energy

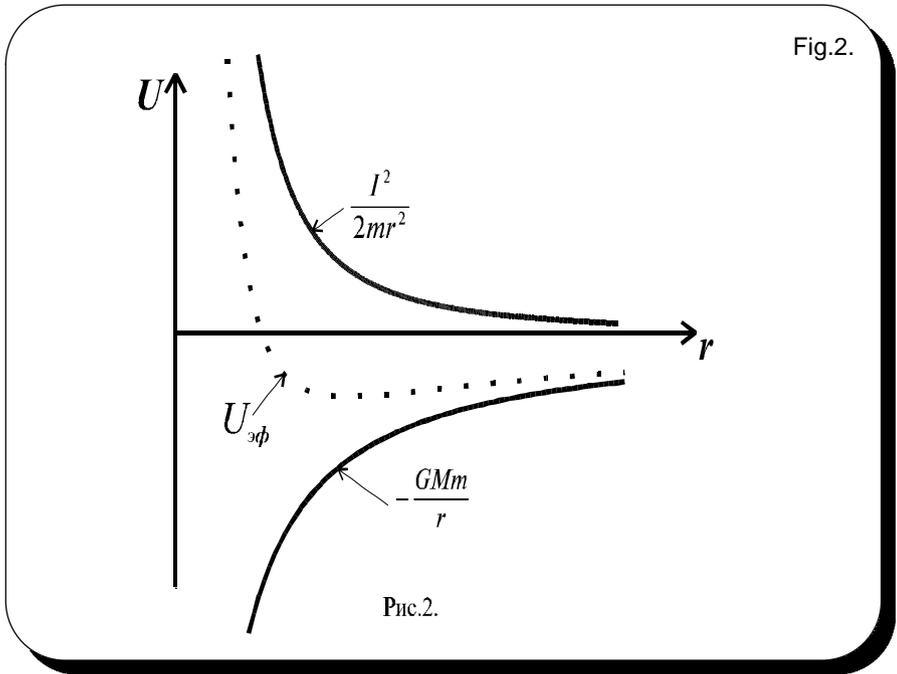
$$U_{ef} = \frac{1}{2} \frac{I^2}{mr^2} - \frac{GMm}{r}.$$

The  $U_{ef}(r)$  depicted in Fig. 2 implies that repulsive forces appear, which prevents the bodies from converging; preservation of the angular momentum brings about a rapid increase of kinetic energy and centrifugal forces as the bodies converge.

If, at a given angular momentum, the total energy (gravitational plus kinetic) of the body is greater than zero, the trajectory of its motion is a hyperbola; if it equals zero, a parabola; and if it is less than zero but greater than  $U_{ef}(r)$  at the minimum point, an ellipse.

The presence of "repulsive" forces makes it possible for bodies interacting in the outer space to exchange energy and impulse as in an elastic collision of atoms in gas.

A dust cloud sustained by proper gravity forces can, under certain conditions, be likened to ideal gas. The velocity of relative motions of particles,  $V$ , in a gaseous body defines the internal energy-that of chaotic motion. The radius of effective



gravitational interaction of particles,  $L$ , can be assessed based on the condition of equality of gravitational and centrifugal acceleration:

$$\frac{GM}{L^2} = \frac{V^2}{L}; \quad L = \frac{GM}{V^2}.$$

Putting the density of particles equal to that of water ( $1 \text{ g/cm}^3$ ) and designating linear dimension of particles as  $l$  gives

$$L = \frac{Gl^3}{V^2}.$$

If the radius of interaction  $L \leq l$ , gravitational attraction of particles will lead to their collision and coalescence. Thus, at a velocity of  $100 \text{ cm/s}$ , the size of coalescing particles will be

$$l = L = \frac{6,67 \cdot 10^{-8} \cdot l^3}{10^4}; \quad l^2 = 0,15 \cdot 10^{12},$$

$$\text{where } l = L = 0,4 \cdot 10^6 \text{ cm} = 4 \text{ km}.$$

For all the particles with  $l < L$ , head-on collision is almost impossible, and their mechanical interaction can be regarded as elastic collision, in which energy is preserved.

Another condition prerequisite for the existence of a gas is intense exchange of energy and impulse between all the particles for distribution of energy over the degrees of freedom. In other words, the free range of particles from collision to collision must be much smaller than the size of the gaseous body. The latter condition is hard to actualize and can take place in an array of rather large cosmic bodies whose size is in excess of many thousand kilometers.

In this range, bodies measuring from 1 to 1000 km can make up clouds, the movement of each body is defined only by the gravity field of the cloud as a whole, and gravitational interaction between the bodies can be ignored. In a sense, the motion of bodies in such a cloud resembles the motion of atoms in a strongly rarefied gas, when the free range is commensurable with, or in excess of, the size of the vessel.

To give an example suitable for quantitative assessments, let us consider parameters of an original cloud of solid bodies, from which the planets of the solar system have formed.

With some rounding, suppose that the mass of the cloud (all the planets)  $M = 2 \cdot 10^{30}$  g and that the moment of momentum of the cloud  $I = 1 \cdot 10^{30}$  g·cm<sup>2</sup>/s. Let us take for granted that, inside the spherical cloud, mass is distributed evenly with a density of  $\bar{\rho}$ .

In this case, the preservation of the moment of momentum makes it possible to relate angular velocity ( $\dot{\Phi}$ ) with dimensions  $R_0$ :  $(2/5)MR_0^2\dot{\Phi} = 2 \cdot 10^{50}$ .

Average kinetic energy of particles in the cloud sustained by the proper gravity field is

$$\frac{MV^2}{2} = \frac{1}{2} \cdot \frac{3}{5} \cdot \frac{GM^2}{R_0},$$

$$V_0 = \sqrt{\frac{3}{5} \cdot \frac{GM}{R_0}}.$$

Assuming  $\dot{\Phi} \approx V_0/R_0$ , the above relationships allow us to estimate both  $R_0$  and  $V_0$ :

$$R_0 = 10^{18} \text{ cm}; V_0 = 300 \text{ cm/s}.$$

In this case, average density in the cloud

$$\bar{\rho} = \frac{2 \cdot 10^{30}}{4 \cdot 10^{54}} = \frac{1}{2} \cdot 10^{-24} \text{ g/cm}^3.$$

Taking the density of matter in particles to be 1 g/cm<sup>2</sup>, let us estimate dimensions of those particles to coalesce on collision:

$$\frac{Gl^3}{L} = V^2; L = l; l^2 = \frac{V^2}{G} = \frac{10^5}{6,67 \cdot 10^{-8}} = \frac{3}{2} \cdot 10^{12},$$

$$\text{where } l \cong 10^6 \text{ cm} = 10 \text{ km}.$$

This is the upper limit. Let us deem this to be the maximal size of particles of that dust cloud from which the solar system was eventually to form.

Size spectrum of particles that arise through segmentation of a solid body can be presented in the form of the distribution:

$$n_i l_i^3 = n_0 l_0^3 \qquad l_0 = l_0(I + i),$$

where  $n_i$  is the number of particles of the size  $l_i$ . Total volumes of particles within each fraction are equal. The number of such fractions is on the order of  $10^2$ ; hence, the size of the smallest particle differs from that of the largest by a factor of  $10^2$ .

$$l_{max} = 100 l_0.$$

In the rarefied initial state, the cloud virtually cannot evolve by itself, as free ranges of particles are greater than the size of the cloud, rendering interaction between the particles negligible.

The probability of collision of particles is proportional to their cross-section  $l_i^2$ , number  $n_i$  and velocity of motion  $v$ .

For each fraction, there is a mass of  $10^{28}$  g; at a density of  $1 \text{ g/cm}^3$ , this yields  $10^{28} \text{ cm}^3$ .

Total cross-section of particles of a size  $l_0 = 10^4 \text{ cm}$  is

$$l_0^2 n_0 = \frac{10^{28}}{10^4} = 10^{24} \text{ cm}^2.$$

A similar calculation for the largest particles yields

$$l_{\max}^2 n_{\max} = \frac{10^{28}}{10^6} = 10^{22} \text{ cm}^2.$$

Consequently, through collisions, the space will be scavenged of fine fractions, and large bodies will increase in size.

The presence of a dust cloud in a certain limited volume will create a gravity field that will pull into the cloud atoms of gases present in the surrounding space. Concentrating in the center of the cloud, atomic gas, under the action of the proper gravity field, will compress and heat up, forming a central heavy body. Under the action of gravity forces of this body, the dust cloud will also begin to compress, and increasing density inside it will intensify the aggregation of particles.

Based on this simplistic scheme, we can time the mass accumulation in the central heavy body.

Given density of atomic gas  $\rho = 10^{-24} \text{ g/cm}^2$  and velocity of gas inflow into the cloud  $v = 3 \cdot 10^2 \text{ cm/s}$ , a mass of the same order of magnitude as that of the Sun will accumulate over a time interval  $T$

$$4\pi R_0^2 \rho v T = 10^{33};$$

$$T = \frac{10^{33}}{4\pi \cdot 10^{36} \cdot 10^{-24} \cdot 3 \cdot 10^2} \approx 0,210^{18} c \approx 10^{10} \text{ years}.$$

This example illustrates how dust clouds, stable per se in the proper gravity field, can initiate an active evolutionary process, forming a gaseous heavy body in the center of the space they occupy.

Further evolution of the cloud itself is determined by the gravity field of the central heavy body, which, with increasing temperature, turns into a star. Scavenging of space of minor bodies and concentration of moment of momentum in large ones brings about a change in the mechanism of its redistribution in the nascent planetary system. Tidal forces, which provide a link between the evolution of the motion of cosmic bodies and formation of their internal structure, become of primary importance.

## Chapter I. Perpetual Motion

Any and all the physical bodies in the space around us—from galaxies to atoms—are in the continuous motion. Celestial bodies connected by gravity forces possess enormous amounts of kinetic energy, which increase as these bodies converge. The energy of the Earth's orbital motion is  $\sim 3 \cdot 10^{40}$  erg, and the rotational energy is  $\sim 3 \cdot 10^{36}$  erg.

For comparison, recall that earthquake energy per one year averages  $\sim 10^{25}$  erg.

An observer on the Earth's surface does not feel this motion. The slow progress of the sun and stars across the sky is not related, in our mind, with the fact that the circumference rotational velocity is about 1000 km/h and that the orbital velocity is  $\sim 30$  km/s.

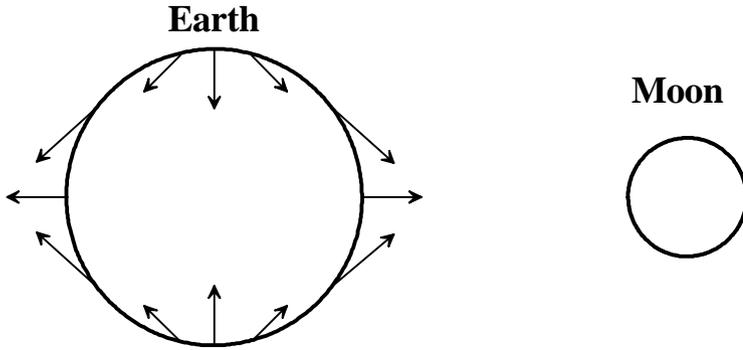
To set in motion and accelerate a massive body, one has to apply a force great enough to overcome cohesion, friction, and inertia. Owing to engineering practice and everyday experience, understanding of this fact has established itself firmly in our consciousness.

On the Earth's surface and in its interior, great masses of fluid and solid media are transferred without apparent effort at the expense of the planet's kinetic energy and gravitational potential. Volume forces of inertia and gravity act upon every particle with a mass in the same manner, irrespective of the particle's position and connections with other particles inside the body. However, the freedom of motion of an individual particle is rather often limited by other particles present. As regards the motion of a finite body as a whole, the resultant of the forces acting on it is equilibrated, in the center of mass, by inertia. For those portions of the body remote from the center of mass such equilibrium is absent, and the body's shape is maintained by internal stresses.

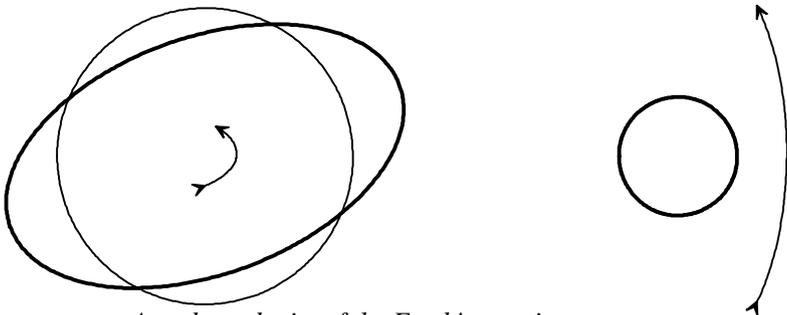
Thus, as the Earth and Moon move about a common center of mass, forces that are created by the Moon's gravity field are equilibrated in the Earth's center by centrifugal force. At all other points of our planet, the acting forces tend to change the Earth's shape. If the Earth rotated about its axis always facing the Moon with the same side, sooner or later masses would be rearranged in it, and its shape would change in such a manner as to counterbalance the lunar attraction forces with the Earth's gravity field. The Earth then would become somewhat elongated along the line connecting the Moon's and the Earth's centers. In Fig. 3, arrows indicate the action of unbalanced forces of lunar attraction upon the Earth's surface.

Under the action of these forces, the Earth is deformed, which does not imply static equilibrium because the Earth's rotation causes the force field to move incessantly. As a result, tidal waves are observed running across the Earth's surface. Regular nearshore changes of sea level were noted as early as the ancient times. Tidal motion in the Earth's crust and atmosphere is also recorded by instruments.

Fig.3.



*Action of tidal forces on the Earth's surface*



*Angular velocity of the Earth's rotation  
is greater than that of the Moon's motion.  
Tidal waves run against the Earth's rotation  
and decelerate the Earth while accelerating the Moon.*

Although the mass motion in the tidal wave is caused by forces that can be predicted with as much precision as needed, this motion is very difficult to quantify because it is not known what constraints are imposed by the structures of the Earth's crust on possible mass displacements. Furthermore, it is not inconceivable that these structures themselves emerged under the action of tidal waves.

One thing is doubtless: that the maximal amplitude of displacement in the tidal wave at any point in the Earth will follow, with some lag, the force-field maximum. Therefore, wave crests on the front and rear parts will be shifted relative to the line that connects the Earth's and Moon's mass centers.

The disfiguring of the Earth's shape in the Moon's gravity field makes it possible for the Earth to transfer some of its angular momentum to the Moon, which

somewhat decelerates the Earth's rotation, while the Moon moves gradually away from the Earth.

The deformational processes and the motion resulting from the tidal waves consume the kinetic energy of the Earth's axial rotation in the amount of  $10^{27}$  erg/yr. The energy thus consumed is channeled in many ways. These include formation of geological structures in the Earth's crust, of oceanic currents, and of atmospheric vortices. The kinetic energy in store, given the present scale of consumption, is virtually inexhaustible. The constancy of the energy flow implies the existence of steady-state energy-consuming structures.

The leading role in the formation of the geospheres (the core, the mantle, the lithosphere, the hydrosphere, and the atmosphere) is played by gravity-induced differentiation of matter by density in the Earth's proper gravity field. At an average density of matter  $\rho = 5,5 \text{ g/cm}^3$ , the difference in density between rocks on the Earth's surface ( $\rho = 2,5 \div 3,0 \text{ g/cm}^3$ ) and in the metallic core ( $\rho \approx 12 \text{ g/cm}^3$ ) is great and testifies to a powerful flow of mechanical energy, which has been maintained by differentiation of matter throughout the Earth's history. It is estimated to be as great as  $\sim 5 \cdot 10^{28}$  erg/yr, a value coinciding with the modern estimates for heat flow on the Earth's surface.

Therefore, the planet's mechanical motion involves enormous energy. This energy gives rise to structures of various rank, through which it is channeled throughout the Earth's volume.

Mechanical motion in the Earth is primordial. The crucial questions in geomechanics are as follows: Why, at each particular hierarchical level, does this motion take a certain steady form? Why do these quasisteady-state motions evolve? What are the mechanisms of distribution of mechanical energy that provide the links between dynamic structures? In all likelihood, it is only by addressing these questions that the knowledge prerequisite for tackling environmental problems is obtained.

Seemingly, structures of the Earth's solid shells are static and cannot exist in harmony with this universal motion. Indeed, properties of the solid, as canonized in the Hookean model, provide no tool for describing the rock-mass flow. But this fact merely leads us to address structured models that allow a better insight into the mechanical properties of solid bodies.

## Chapter II. Structures of Solid Body

### 1. Dynamic Structure of Real Elastic Body

In an ideal elastic body, changes of shape are resisted by inner stresses that increase linearly with deformation. Deformation ceases as soon as the inner stresses have completely counterbalanced the action of external forces. An ideal elastic body is very strong and indestructible. It lacks mechanisms to absorb mechanical energy; hence, vibration once excited is never damped.

In real elastic bodies, all the features just mentioned are, strictly speaking, missing. Nonideality of bodies is related with defects of the atomic structure of crystals (dislocations and fractures). This accounts for the low yield stress, plastic flow, local discontinuities, and absorption of macrovibrational energy. Crystal lattice, at the level of atomic vibrations, is capable of absorbing a great amount of mechanical energy of macromotion without introducing significant changes into mechanical properties of solid bodies.

The necessity to consider structural parameters of solid has long been evident. All sorts of corrections and limitations regarding the applicability of the ideal-solid model were introduced: elastic limit, absorption indexes of elastic vibration, yield stress under static and dynamic load, etc. This kind of approach to reconciling the model and the real body proved rather effective in mechanical engineering, as all constructions work within the limit of minor, elastic, deformations. However, in addressing rocks and rock massifs, it is easy to detect a tremendous variety of defects and inhomogeneities. Research and classification of structural peculiarities of rock massifs have been the subject of many studies.

The diversity and broad scale range of the structures seemingly doomed to failure any attempt to find their generalized image not overloaded with a multitude of empirical information. That this difficulty was overcome was favored by the fact that, under homogeneous, constant-velocity deformation, only a small part of defects in a solid body are actively involved in the process, while the rest do not manifest themselves in any manner and, consequently, cannot bear on the mechanical properties of the solid body. Needless to say, as the velocity and kind of deformation change, the structure of mechanically significant inhomogeneities also changes. It became a necessity to elaborate a procedure for determining the parameters of mechanically significant structures in real solids and to learn to model such structures.

By developing, phenomenologically, the idea of defectiveness of crystal lattice, it became possible to construct a solid-body model that accounted, in general terms, for structural features of real solids.

Let us clarify the main concepts of this model. Variably sized spherical inhomogeneities are implanted in an isotropic elastic Hookean body. Inhomogeneities of each particular size are distributed in the body evenly, but not necessarily regularly. Size distribution of the inhomogeneities is chosen so that bodies of different size have a similar structure:

$$\frac{l^3 dn}{d \ln l} = \alpha,$$

where  $l$  is the size of inhomogeneities,  $n$  is the number of inhomogeneities of the size  $l$  in a unit of volume, and  $\alpha$  is a constant characterizing the defectiveness of the solid.

This formula asserts that, in the body, total volumes of inhomogeneities of each particular size are equal and occupy a minor portion of the space, which is designated  $\alpha$ .

Sometimes it is convenient to use inhomogeneities, the sizes of which present a discrete series, such as

$$\frac{l_{i+1}}{l_i} = 2^i.$$

In this case, the formula for size distribution of inhomogeneities assumes the form  $l_i^3 n_i = \alpha$ , where  $i$  takes on all the values of the numerical series, positive and negative. The reader must not be confused by the fact that the total volume of inhomogeneities can exceed that of the body because minor inhomogeneities are included in major ones and comparison with the body's volume has no sense.

What mechanical properties must be attributed to these inclusions in the elastic body, which were designated as inhomogeneities? To contrast with the elastic body, the "inhomogeneities" must provide an irreversible change of form under stresses as small as is wished. On the other hand, this capacity of the inhomogeneities must be regulated by the deformational process in the body to the point where elastic properties at the "inhomogeneity" are restored. Such mechanical behavior of an inhomogeneity can be actualized if inside the inhomogeneity, which is deformed like the whole solid body, excessive shear stresses arise (the volume of the inhomogeneity remains unchanged during deformation) and become weaker (relaxed) by themselves with time. At the level of crystal lattice, such phenomena in the neighborhood of a defect can be physically substantiated; in our model, these are postulated for all the scale levels.

Because relaxation of stresses occurs in time, it is convenient to choose the velocity of deformation ( $\dot{\epsilon}$ ) for a regulating parameter.

The two competing processes-concentration of excess stresses and their relaxation at a chosen deformation velocity of the body-defines stresses at the inhomogeneities and permits identifying the structure significant for the characterization of mechanical properties of the solid body. Pertaining equation has the form

$$\frac{d\Delta\sigma_l}{dt} = \rho c_l^2 \dot{\epsilon} - \nu \frac{\Delta\sigma_l}{l};$$

where  $\Delta\sigma_l$ , is excess stress at  $l$ -size inhomogeneity;

$\rho c_l^2 = \mu$  is the product of density of the body by velocity of transverse waves in it, by definition equal to shear modulus;

$\dot{\epsilon}$  is shear strain velocity in the solid body;  $\nu$  is stress-relaxation velocity, whose value for all the rocks is assumed at  $2 \cdot 10^6$  cm/s.

Integrating this equation for a constant deformation velocity, we obtain values for stresses on inhomogeneities of different sizes:

$$\Delta\sigma_l = \rho c_t^2 \dot{\epsilon}_v \frac{l}{v} [1 - e^{-vt/l}].$$

Needless to say, deformation at a constant velocity advances over a limited time interval, until the elastic limit of the body is attained. Deformation of natural large-scale bodies has a minor velocity of  $\sim 10^{-5} \div 10^{-7}$  1/yr, hence the interval of time in which the velocity can remain constant ranges from 100 to 10 000 years. In practice, we never know when that deformation process started, which affects at present the massif in question. Setting the time  $t$  arbitrarily, let us define a certain value  $l^* = vt$ , which divides all the inhomogeneities into two groups:

for  $l < l^*$ ,

$$\Delta\sigma_l = \rho c_t^2 \dot{\epsilon}_v \frac{l}{v}.$$

Stresses at minor heterogeneities, due to continuous relaxation, remain at the same level: the larger the inhomogeneity, the greater the stresses.

For  $l > l^*$ ,

$$\Delta\sigma_l = \rho c_t^2 \dot{\epsilon}_v \frac{l}{v} [1 - (1 - \frac{vt}{l})] = \rho c_t^2 \dot{\epsilon}_v t.$$

Stresses at large inhomogeneities have no time to relax and grow with time in the same manner as in the whole body.

It follows that structure of mechanically significant inhomogeneities changes with time: the prevalent size of inhomogeneities  $l^* = vt$ . At minor inhomogeneities ( $l \ll l^*$ ), there are no stresses; at major ( $l > l^*$ ), there is no relaxation.

In an ideally elastic body, stresses are uniquely related with deformation. Among the determining parameters, there are no characteristic values with dimensions of length, and for this reason geometrically similar bodies have the same stress-strain state. In real structured bodies, similitude will take place only when the dynamic structures are similar. Recall that the hierarchy of inhomogeneities in a solid, in agreement with the assumed formula of their distribution, provides for geometrical similitude of variously sized bodies.

As for the dynamic structure of stressed inhomogeneities, this depends on deformation velocity (dimension of  $[\dot{\epsilon}] = s^{-1}$ ). Considering that among those parameters characterizing mechanical properties of the body there appears a relaxation velocity  $v = 2 \cdot 10^{-6}$  cm/s, the condition of geometrical similitude of structures in such bodies

$$\frac{L\dot{\epsilon}}{v} = const; L - \text{body size.}$$

Hence, similitude of the mechanical state of two bodies of different sizes  $L$  will occur if the bodies are themselves geometrically similar, if deformation velocities in them are inversely proportional to their sizes ( $\dot{\epsilon}_I/\dot{\epsilon}_{II} = L_{II}/L_I$ ), and if the time elapsed since the beginning of deformation in them is proportional to their sizes ( $t_I/t_{II} = L_I/L_{II}$ ).

Note another dimensionless parameter  $\alpha$ , which equals the total volume of inhomogeneities of the same size divided by the body's volume. The volume of elastic body per one inhomogeneity of a given size is defined by the relationship  $a_i^3 = l_i^3 / \alpha$ , where  $a_i$  is distance between inhomogeneities. Accordingly, distances between inhomogeneities  $l^*$  will equal  $a^* = l^* / \alpha^{1/3}$ . It is worthwhile to compare dimensions of the body  $L$  and  $a^*$ .

In relatively small bodies that are deformed at a small velocity ( $\dot{\epsilon} = \text{const}$ ), the  $a^*(t)$  growing with time can exceed the body's size  $L$ . This means that at all the inhomogeneities stresses have stopped growing, and the dynamic structure will be of little importance in the further deformation.

To this we can only add that excess stresses at inhomogeneities will persist not only throughout the deformation process but also when it has ceased, although then they will decrease gradually. It should be emphasized that these stresses will not disappear even when the body is completely relieved of any load.

The hierarchy of various-scale inhomogeneities manifests itself in mechanical properties of solid bodies, which have a paradoxical aspect.

Damping of elastic waves can be characterized by the ratio of amplitudes at the ends of a path segment equal to the wavelength  $l$ . Usually, damping is described exponentially:

$$A = A_0 \exp(-\gamma x) f(x - ct); \quad c - \text{elastic-wave velocity.}$$

The logarithm of the amplitude ratio on the path segment  $l$  is termed logarithmic decrement  $\delta$ :

$$\delta = \lg \frac{A}{A_0} = \gamma l = \gamma \frac{c}{A}.$$

The energy of the wave is proportional to the square of the amplitude; hence, the energy loss by the wave over the segment  $x = l$ :

$$\frac{\Delta E}{E} = 2\sigma = 2\gamma \frac{c}{A} = \frac{2\pi}{Q}.$$

The merit of material is termed the  $Q$  factor; hereinafter, this parameter will be used as a characteristic of rocks.

Experiments show a striking property of rocks: the damping decrement (or  $Q$  factor) in them does not depend on the vibration frequency. This means that energy absorption in the propagation of variously lengthed elastic waves will be equal along paths proportional to the wavelengths. In terms of the continual solid model, without defects or with equally sized defects, this property cannot be explained. The point is that length-variable elastic waves appear to choose a suitable, in terms of scale, structure to uniquely interact with in the solid body.

The dynamic structures discussed above and arising in a solid body with inhomogeneities under strain explain this property of rocks. Appropriate mathematical operations, which are omitted here, reveal a relationship between the parameters adopted above for the solid-body model with inhomogeneities, and the empirical merit coefficient  $Q$ . The merit of material proved to be uniquely defined by the total volume of inhomogeneities of the same size  $\alpha$ :

$$\frac{l^3 dn}{d \ln l} = \alpha = \frac{2}{\pi Q}.$$

Therefore, the hardy task of establishing the distribution of mechanically significant inhomogeneities in real bodies found its solution in a simple, long-known procedure of defining the  $Q$  factor of the material.

It is worthwhile to mention here the difference in scales of the structures responsible for absorption of elastic energy and for the scattering of elastic waves. Elastic waves are most effectively dissipated on inhomogeneities that are commensurable in size with the wavelength. As for the inhomogeneities on which mechanical energy is dissipated, their size  $l$  must be on the order of  $l^* = \nu T = \nu \frac{\lambda}{c}$ , i.e., ca.  $10^{11}$  times smaller. Thus, an elastic wave with a wavelength of  $\sim 500\text{m}$  ( $f=10\text{Hz}$ ) will dissipate its energy on inhomogeneities  $l^* \sim 10^{-6}\text{ cm}$ .

It follows that seismic vibrations, which must be ubiquitous at any time, are capable of activating processes in rocks at the microlevel.

## **2. Disintegration Structure of Solid Body**

Disintegration that takes place in the solid body, as this is homogeneously deformed, is characterized by the average size of a clast (block, parting) and by clast-size distribution. The formation of a multitude of size-variable clasts in the solid body, when deformation at each particular point is the same, appears to be a rather complicated process. At least two variants for its actualization can be suggested.

During a slow development, randomly appearing sporadic fractures cut the body into large blocks that are crushed into smaller ones, as the deformation grows. In this case, the energy required for disintegration must be supplied from the outside into the volume being deformed.

In a fast deformation, considering that disintegration is defined by the velocity of fracture propagation, which is smaller than the velocity of longitudinal waves in an elastic body, elastic energy can accumulate in the body in the amount sufficient for the body to become finely crushed. In this case, the body develops numerous embryonic fractures that propagate to intersect randomly, forming size-variable clasts. Mechanical-energy supply from the outside at this instance can be ignored. It is natural to assume that each variant of disintegration will be distinguished by a specific clast-size distribution.

Experiments show that, indeed, sequential crushing of clasts in a ball mill (rock is crushed by metal balls in rotating drums) yields a log-normal clast-size distribution and that crushing in a blast wave yields Weibull's distribution.

The average clast size for both variants of disintegration depends on the energy expended on crushing per unit of volume: the greater the energy, the smaller the average clast size. The attempts to establish quantitative relationships based on the idea of identity of the crushing energy and the energy consumed to form a new surface proved a failure both due to the methodical difficulty of assessing the total surface of the newly formed fine particles and owing to the uncertainty of the value of the surface energy of real bodies.

Inhomogeneities in natural bodies are of primary importance in defining the size of clasts formed through crushing; therefore, the dependency of the average-sized

clast on the energy consumed by crushing must be sought in the change of dynamic structure of the stressed state of the body being deformed. These changes are defined by the velocity of deformation or, in other words, by the amount of mechanical energy received by the body in a unit of time. As was mentioned in the previous section, stresses at inhomogeneities initially grow with strain, and then stop growing at a value which is proportional to the size of the inhomogeneity:

$$\Delta\sigma_l = \rho c_i^2 \dot{\epsilon} \frac{l}{v}.$$

At all the inhomogeneities, where  $\Delta\sigma_l$  will exceed the yield strength  $\sigma^*$ , embryonic fractures will form, whose subsequent propagation will be defined by the amount of elastic energy in the disintegrating body. Let us designate the size of the smallest of such inhomogeneities as  $l_0$ :

$$\sigma^* = \rho c_i^2 \dot{\epsilon} \frac{l_0}{v}.$$

The distance between embryonic fractures in the solid body  $L_0$  will define the minimum size of the clast, and, consequently, its average size, because clast-size distribution is taken to be known. Recall that  $L_0$  and  $l_0$  are related by the formula

$$\frac{L_0^3}{l_0^3} = \frac{\pi Q}{2}, \text{ where } Q \text{ is the merit of the material.}$$

Based on experiments on clast-size distribution, a single-stage crushing of a solid body results in a particle-size distribution that can be presented in the form that can be given a simple and clear interpretation:

$$L_i^3 N_i = L_0^3 N_0; L_i = iL_0.$$

All the fractions of particles of sizes divisible by  $L_0$  occupy the same volume. This situation somewhat resembles the law of equal energy distribution by degrees of freedom: the disintegration energy is distributed evenly between the fractions.

The size distribution itself differs qualitatively from the distribution of inhomogeneities in the solid body prior to disintegration because the characteristic length  $L_0$  has appeared, which provides the scale for measuring the clasts.

The number of the fractions is determined by the probability of formation of a clast of the smallest size  $L_0$ . Fractures at neighboring inhomogeneities  $l_0$  very seldom happen to intersect so as to produce an isolated volume  $L_0^3$ . Estimates show this probability to be on the order of 0.01. Hereinafter, for the sake of accuracy, we will use a probability of  $1/3^4$ . Hence, the whole crushed mass falls evenly into  $3^4 = 81$   $L_0$ -wide fraction intervals. The maximum clast size,  $L_{max} = 81L_0$ , and, accordingly, the average size in the distribution is

$$\frac{L_{max}+L_0}{2} \approx 40L_0.$$

The extremely broad spectrum of particles resulting from crushing requires large volumes of solid bodies to be studied in order to obtain reliable distribution parameters. Thus, the full spectrum of particles, including at least one clast of the maximum size, has a total volume  $V_0 = 81L_{max}^3 = 0,4 \cdot 10^8 L_0^3$ . Therefore, the

starting size of the body under study must exceed the minimum clast size by a factor of almost 1000:

$$\sqrt[3]{V_0} \geq 350 L_0.$$

This entails the limited capabilities of the type tests for studying the dependency of grain-size distribution on the scale and velocity of deformation. Experimental data on grain-size distribution for an arbitrarily limited volume (e.g., sample) distort not only the scale  $L_0$  but also the character of the clast-size distribution.

Whenever it is necessary to compare the effects of disintegration of a solid body, disturbances of continuity, and related parameters at different scale levels, one should keep in mind that mechanical similitude includes not only the geometry of the body but also its dynamic structures.

Hierarchical arrangement of the block structure of natural bodies imparts to them new qualities: this not only facilitates deformation under volume forces but also records the sense of displacements that have occurred. This mechanical memory can play the role of inertia during slow motion.

The ubiquity of block structure in rock massifs and the broad occurrence of "live" fractures in the Earth's crust testify that dynamic structures are a form of self-organization of motions in the Earth's solid shells.

The mobility of solid bodies with block structure is due to the fact that deformation involves both displacement of blocks relative to each other and deformation of the blocks themselves. In block-structured bodies, deformation gives rise to chains of blocks that carry the load and form within the body a framework that resists the change of the body's form. Another group of the blocks remain only slightly stressed or entirely free of any load. The rearrangement of the framework structure is accompanied by dynamic processes that excite elastic vibrations throughout the body.

Free blocks that remain mobile can jostle inside the block structure under the action of elastic waves propagating within the body. These shifts, unrelated to the deformation of the whole body, prepare a rearrangement of the framework structure-redistribution of load between blocks. In a sense, their role in the deformation process is similar to that of dislocations that arise when a crystal lattice is being deformed.

### **3. Slow Motion in Solid Bodies**

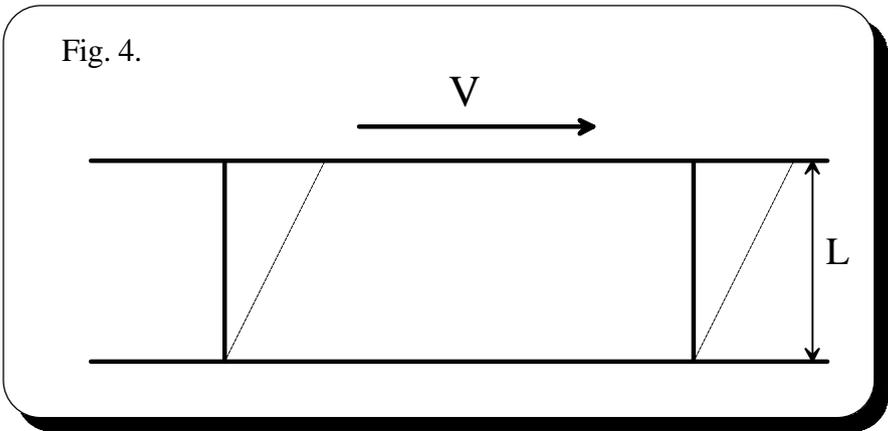
Many materials possess plasticity and are easily deformed without losing continuity. Plastic deformation usually takes place when the material is compressed by external forces that prevent fracturing. As a rule, the plasticity of material is of interest to the student of mechanics if stresses in the solid body exceed the elastic limit. Plastic flow, however, is also known to take place at low stresses in the solid body, provided the loading lasts for a very long time.

Such a behavior of the solid body is a manifestation of its dynamic structure. As a solid body is being deformed at a constant velocity, a constant excess stress develops at minor inhomogeneities; this stress is proportional to the deformation velocity and the linear size of inhomogeneity.

If the body has finite dimensions, in time, stresses will cease growing at all inhomogeneities because of their relaxation. In a large but limited body, inhomogeneities are also limited in size; hence, by reducing the deformation velocity, the excess stresses can be made as small as is wished. From standpoint of the balance of internal and external forces, the stressed inhomogeneities under slow deformation can be, seemingly, ignored, but let us consider the other aspects of the situation in question. In a large body, the total volume of inhomogeneities of all sizes can exceed the body's proper size. Of course, a simple addition of volumes of size-variable inhomogeneities yields but an approximation. The lack of excess stresses results from the relaxation within each particular inhomogeneity. It is as if these inhomogeneities are fluid inclusions in the solid. As long as their number is small, the solid body retains its starting properties, but, when a certain concentration of inhomogeneities is attained, these can merge and divide solid blocks by "fluid" interlayers. Naturally, mechanical properties of the body begin to change qualitatively. A similar phenomenon is observed, when a gas-saturated fluid starts boiling on a rapid decompression: fluid with gas bubbles turns into gas with fluid drops; these two structures are completely different.

Hence, as a large body with a low  $Q$  factor is deformed, the increasing concentration of inhomogeneities under a constant stress brings about a transformation in the dynamic structure: the change in the body's shape is now resisted by stresses at the inhomogeneities, and elastic stresses can be ignored. In this manner, creep is interpreted in terms of the structured solid body model. An unlimited solid medium at any given deformation velocity is granted to contain an inhomogeneity large enough to concentrate stresses that will grow to the point of destruction; hence, irreversible deformations without a breach of continuity in an unlimited medium are impossible.

Our discussion of dynamic structure under slow deformation of bodies so far has ignored the qualitative criteria of the appearance (or existence) of the creep mode.



Let us consider the simplest case of shear deformation of a body, illustrated in Fig. 4. The two flanks of the strip are displaced relative to each other at the velocity  $V$ .

The deformation velocity inside the  $L$ -wide strip  $\dot{\epsilon} = V/L$ . The stress at the inhomogeneities is

$$\Delta\sigma_l = \rho c_t^2 \dot{\epsilon} \frac{l}{v} = \rho c_t^2 \frac{Vl}{vL}; \quad v = 2 \cdot 10^{-6} \text{ cm/s.}$$

The creep mode is possible if  $\Delta\sigma_l \ll \sigma^*$ . Taking into account that strength characteristics of rocks are usually no greater than  $10^{-3} \rho c_t^2$ , this gives

$$\frac{\Delta\sigma_l}{\rho c_t^2} = \frac{Vl}{vL} \ll 10^{-3}, \text{ or } V \ll 10^{-3} v \frac{L}{l}.$$

Taking the distance between the largest inhomogeneities to be on the order of the stripe's width  $L$ , the relationship  $\frac{L}{l} = \left(\frac{\pi \cdot Q}{2}\right)^{1/3}$  is a constant. It follows that

$$V < 10^{-3} v \left(\frac{\pi \cdot Q}{2}\right)^{1/3}.$$

Therefore, the creep mode is possible if the displacement velocity at the stripe's flanks does not exceed a certain critical value  $V^*$  that depends on the merit of the material. At  $Q = 100$ ,  $V^* = 0.3$  cm/yr. Note now that the criterion we have introduced does not include the size of the body under deformation. Hence, mechanics of bodies deformed in the creep mode is slow-motion, rather than slow-deformation, mechanics. Two similar but differently sized bodies will have a similar structure if the velocities at which their boundaries are displaced are the same. In this case, the deformation velocities of the smaller body will be greater with respect to the bodies' dimensions, and inhomogeneities with equal stresses will have sizes proportional to those of the bodies.

If such similitude of structures takes place, nonelastic stresses in the stripe being deformed will also be equal. This, in its turn, means that energy expenses for displacement of bodies separated by such a stripe do not depend on its width  $L$ .

This indifference to the width of boundary zones under slow motion of solid bodies renders this flow qualitatively different from the viscous-fluid flow, in which shear stress depends on the velocity gradient  $V/L$ . Recall that, in fluid and in gas, viscous forces are related with transmission of momentum from layer to layer. In a solid body, nonelastic stresses result from stress relaxation at inhomogeneities. The likeness of creep and viscous fluid flow arises on the basis of fundamentally different mechanisms of mechanical-energy dissipation.

The perpetual motion, which involves all the physical bodies from galaxies to atoms, is maintained by the flow of mechanical energy from major bodies (which possess enormous gravity potentials) to ever smaller bodies. Therein lies the function of dynamic structures in the geospheres, including the solid shells.

## Chapter III. Problems and Solutions

### 1. Paradoxes of Disintegration of Solid Body

Let us consider stress structures in a solid body that appear during loading and unloading of a cylindrical column.

Let us write the equation for excess stresses at inhomogeneities in a tensor form:

$$\frac{d\Delta\sigma_{ik}^l}{dt} = \rho c_t^2 (\dot{u}_{ik} - \frac{1}{3}\dot{u}_{ll}\delta_{ik}) - v \frac{\Delta\sigma_{ik}^l}{l}$$

where  $i, k$  designate axes  $(x, y, z)$ ;  $(u_{ik} - \frac{1}{3}u_{ll}\delta_{ik})$  is shear deformation;

$$\delta_{ik} = \begin{cases} 1 & \text{at } i=k \\ 0 & \text{at } i \neq k \end{cases}$$

$\dot{u}_{ik}$  is deformation velocity. The excess stress, which brings about a volume change, is absent at inhomogeneities:

$$\Delta\sigma_{ll} = \Delta\sigma_{xx} + \Delta\sigma_{yy} + \Delta\sigma_{zz} = 0.$$

That portion of the body's volume occupied by inhomogeneities of the same size  $l$  is defined by the merit of material:

$$\frac{l^3 dn}{dln \cdot l} = \frac{2}{\pi Q}, \text{ where } n \text{ is the number of inhomogeneities in the unit of volume.}$$

Average nonelastic stresses in the solid body are defined by the sum total of stresses at inhomogeneities, with due account for the proportion of area that these occupy:

$$\sigma_{ik}^l = \frac{2}{\pi Q} \int_l \sigma_{ik}^l dln l.$$

Suppose the loading of the column over time is given:

$$P(t) = \begin{cases} P_T^l, & 0 < t < T \\ P & t > T \end{cases}$$

Deformation of the column (the  $Z$  axis is vertical) is ambiguously related with the load value  $P$ :

$$u_{zz} = \frac{P}{\pi a^2 E} - \frac{\sigma_{zz}^l}{E},$$

where  $E$  is Young's modulus;  $\pi a^2$  is the cross-section of the column; and compressional forces  $(P, \sigma_{zz})$ , like the compressional strain, are negative values.

The nonelastic stresses  $\sigma'_{zz}$  depend on the velocity of deformation, which must be found by solving the problem. Considering that the nonelastic stresses are much smaller than elastic, one can use the following methods:

the iterative method,

$$u_{zz} \approx \frac{P(t)}{\pi a^2 E} = \frac{P}{\pi a^2 E} t,$$

$$\dot{u}_{zz} \approx \frac{P}{\pi a^2 E} \cdot \frac{1}{T} \text{ in the range } 0 \leq t \leq T;$$

the elastic deformations,  $u_{xx} = u_{yy} = -\sigma u_{zz}$ ;  $\sigma$  is Poisson's coefficient;

the volume deformation,  $u_{ll} = u_{xx} + u_{yy} + u_{zz} = (1 - 2\sigma)u_{zz}$ ;

the shear deformation,  $(u_{zz} - \frac{1}{3}u_{ll}) = \frac{2}{3}(1 + \sigma)u_{zz}$ .

Let us substitute in the equation for stresses at inhomogeneities the velocity of shear strains  $\frac{2}{3}(1 + \sigma)\dot{u}_{zz}$  and replace

$$\rho C_t^2 = \mu = \frac{E}{2(1+\sigma)}. \text{ The result is}$$

$$\frac{d\Delta\sigma'_{zz}}{dt} = \frac{E}{3}\dot{u}_{zz} - v\frac{\Delta\sigma'_{zz}}{l}.$$

Integrating over time interval  $0 < t < T$ , when  $\dot{u}_{zz} = \frac{P}{\pi a^2 E T}$  gives

$$\Delta\sigma'_{zz} = \frac{P}{3\pi a^2 v T} [1 - e^{-vt/l}].$$

The other components of the tensor are

$$\Delta\sigma_{xx} = \Delta\sigma_{yy} = -\frac{1}{2}\Delta\sigma_{zz}.$$

Note that stresses normal to the side of the column have a different sign:

if  $\sigma_{zz}$  is compression stress,  $\sigma_{xx} = \sigma_{yy}$  is tensile stress.

In a simplified form, the dependency  $\Delta\sigma'_{zz}(T, l)$  for the instant of time  $t=T$  can be written as

$$\Delta\sigma'_{zz} \approx \frac{1}{3} \frac{P}{\pi a^2 v T} \text{ for } \frac{l}{vT} < 1,$$

$$\Delta\sigma'_{zz} \approx \frac{1}{3} \frac{P}{\pi a^2} \text{ for } \frac{l}{vT} > 1.$$

At major inhomogeneities, excess stresses are equal and are as high as 1/3 of the elastic stresses in the column. At minor inhomogeneities, stresses are proportional to  $l$  because, while integrating, we will ignore their contribution to the value of nonelastic stresses:

$$\sigma'_{zz} = \frac{2}{\pi Q} \int_{l=vt}^a \Delta\sigma'_{zz} dl \ln l = \frac{P}{\pi a^2} \frac{2}{3\pi Q} \ln \frac{a}{vT}.$$

Let us substitute values that could be encountered in practice,  $a = 100\text{cm}$ ,  $T = 10^5\text{s}$ ,  $Q = 10^2$ :

$$\sigma'_{zz} = \frac{P}{\pi a^2} \frac{2}{3\pi \cdot 10^2} \ln \frac{100}{2 \cdot 10^{-6} \cdot 10^5} \approx 10^{-2} \frac{P}{\pi a^2} .$$

It is clear that nonelastic stresses are, indeed, very small; therefore, the situation is seemingly static in all cases, unless the forces of inertia cannot be ignored. This conclusion, however, is premature. The matter is that nonelastic stresses even after the loading of the column continue to change with time; hence, the deformation of the column will also change. It would be incorrect to merely compare the values of elastic and nonelastic stresses without considering the temporal parameters.

At a constant load  $P$ , the velocity of deformation effectively equals zero, giving

$$\frac{d\Delta\sigma'_{zz}}{dt} = -v \frac{\Delta\sigma'_{zz}}{l} .$$

Integrating over time, from the point where the excess stresses at inhomogeneities have attained  $\Delta\sigma_{zz} = \frac{1}{3} \frac{P}{\pi a^2}$  and will continue to relax, gives

$$\Delta\sigma'_{zz} = \frac{P}{3\pi a^2} e^{-vt/l} .$$

To determine the nonelastic stresses, one must integrate over all the inhomogeneities. Let us use a simplified form of the integrated function.

$$\begin{aligned} \sigma'_{zz} &= \frac{2}{\pi \cdot Q} \cdot \frac{P}{3\pi a^2} \int_{l^*}^a e^{-vt/l} d \ln l; \quad l^* = v(T + t) \\ e^{-vt/l} &\approx \begin{cases} 1; at; vt/l < 1 \\ 0; at; vt/l > 1 \end{cases} \\ \sigma'_{zz} &\approx \frac{2}{\pi \cdot Q} \cdot \frac{P}{3\pi a^2} \ln \frac{a}{v(t+T)} . \end{aligned}$$

Although, at inhomogeneities of each particular size, stresses relax exponentially, the nonelastic stresses decrease more slowly, following a logarithmic law:

$$\frac{\sigma'_{zz}}{(\sigma'_{zz})_{t=0}} = \frac{\ln(a/vt)}{\ln(a/vT)} \quad (t \gg T) .$$

The time interval over which a nonelastic stress decreases by a factor of two, as follows from the last relationship, will be one order of magnitude greater than the loading time  $T$ . Thus, in the case just given,  $T = 10^5$  s (twenty-four hours). This means that the deformation of the column due to relaxation of nonelastic stresses will take several decades. The long duration of manifestation of the dynamic structure can substantially change the estimated role of nonelastic stresses when the case in point is the schedule and terms of construction of a major structure.

Let us now consider the stress structure in a column that is loaded after relaxation of all the stresses at inhomogeneities. If the unloading takes the time  $T$  following a linear law, stresses at inhomogeneities appear again, but with a different sign:

$$\Delta\sigma_{zz}^l \approx -\frac{1}{3}\frac{P}{\pi a^2} \quad (\text{at inhomogeneities } l > \nu T).$$

Stresses at inhomogeneities have the same sign as the deformation: under compression,  $u_{zz} < 0$  and  $\Delta\sigma_{zz} < 0$ ; under extension,  $u_{zz} > 0$  è  $\Delta\sigma_{zz} > 0$ .

As the column was being loaded,  $\sigma_{zz}^l$  resisted the loading. As nonelastic stresses were relaxing, elastic forces began to bear the load, due to which the column continued to compress.

During the release, nonelastic forces resist the extension, which takes place as the column is restored to its initial length by elastic forces. When the load has been completely removed, elastic stresses in the column will not disappear entirely because they must counterbalance the action of tensile stresses at inhomogeneities.

In an ideal solid body under slow loading and unloading (the duration of which, multiplied by the elastic-wave velocity, is much greater than the body's size) strains strictly follow the stresses, i.e., are related by Hooke's law. In our case of loading and unloading of the column,  $u_{zz} = \frac{P}{\pi a^2 E}$  and  $u_{zz} = \frac{\sigma_{zz}}{E}$ .

If  $\sigma_{zz}$  does not exceed the crushing strength of the material, no discontinuities must appear in the column, as tensile stresses are absent. However, rock pillars that support the roof of underground excavations (analogues of the columns we have discussed) display vertical fractures and exfoliation of side surfaces suggesting tensile stresses.

For a homogeneous elastic body, such a response to axial load seems to be paradoxical and inexplicable. Inhomogeneities, at which excess stresses appear on loading, make it possible to relate this phenomenon with the dynamic structure of the solid body.

If during loading of a column stresses at inhomogeneities  $\Delta\sigma_{zz}$  are compressive, stresses  $\Delta\sigma_{yy} = \Delta\sigma_{xx}$  are tensile:

$$\Delta\sigma_{xx} = -\frac{1}{2}\Delta\sigma_{zz} = -\frac{1}{2}\frac{P}{3\pi a^2} = \frac{1}{6}\sigma_{zz}.$$

Although these stresses are notably lower than those counterbalancing the axial load, they act on areas where elastic stresses are totally absent, and tensile strength of rocks often is one order of magnitude lower than the crushing strength. Therefore, the dynamic structure plays a decisive role in the initiation of fractures, the growth of which brings about disintegration of the solid body.

Still more paradoxical appears the solid body disintegration on unloading, when compressional elastic stresses in the body decrease. In this case, too, the dynamic structure of stresses at inhomogeneities accounts for the appearance of fractures in the column's cross-section during unloading-tensile stresses at inhomogeneities are as strong as one-third of the compressive elastic stresses and, as was demonstrated, do not disappear at once, as forces that compress the body diminish.

Both paradoxical effects can be reproduced in experiments on destruction of brittle bodies, and the results can be matched with the estimates derived from the concepts of the solid-body model.

## 2. Seismicity and Disintegration Structure

Among the plentiful evidence for a continuous deformation in the Earth's interior, earthquakes are likely to be of utmost importance. In human perception, earthquakes are a terrifying manifestation of natural forces.

The systematic effort in creating the network for instrumental recording of earthquakes and in storing the data has produced, over the last century, remarkable results. Earthquakes were found to show a certain energy distribution, which was formulated as the Gutenberg-Richter law of earthquake recurrence. Seismic vibrations of the Earth's surface due to elastic waves emitted through earthquakes provide the main insight into the Earth's internal structure. The very earthquake focus, however, still presents an extreme difficulty for investigation. This is due to the fact that elastic waves are formed outside the focus and can thus furnish only a very general idea of the character of motion in the source zone.

According to Yu.V. Riznichenko, who put forward the idea of continuity of the seismic-energy flow, the key problem in defining the seismic zoning is establishing the strongest earthquake possible for one or another region. The Gutenberg-Richter law establishes the frequency of recurrence of earthquakes with different energies. Earthquake energy is directly proportional to the volume in which the accumulated energy of elastic stresses in a solid body brings about a fast change in this body's shape, accompanied by elastic-wave emission. Proceeding from energy to the volume of the source  $L^3$  gives a very simple expression for the earthquake-recurrence law:

$$L_i^3 N_i = \text{const}, \text{ where } L_i = iL_0 (1 < i < 81), \text{ and}$$

$N_i$  is the number of earthquakes per year.

This law was established for the Earth as a whole for energy range of  $10^{18}$  erg to  $10^{24}$  erg.

Later, such dependencies were also established for specific regions. In this case, the ranges of earthquake energy were not infrequently limited by the sensitivity of instruments used. As a result, the characteristic size of the source, which could be related with the natural range of earthquake energies in the region, remained beyond the scope of study.

Earthquake foci are unevenly distributed in space, and zones of seismicity are limited. A limited volume cannot produce an earthquake whose focus will be commensurable with it. Purely geometrical considerations, however, can only provide a basis for limiting values, which may never occur in reality.

Let us make use of the fact that the Gutenberg-Richter law formally coincides with the clast-size distribution in a solid body being disintegrated. The minimal and maximal clast sizes depend on the velocity of deformation of the body. Suppose that an earthquake consists in that a block is formed and separated from the surrounding rock massif. It is only in this case, similar to a string snapping, that a fast change in the shape can take place, giving rise to elastic waves in the surrounding space. Half the elastic energy stored in the source will emit outward,

and the other half will be "lost" in the damping vibrations of the isolated block. The source sizes of minimal and maximal earthquakes are related like those of clasts in a body being disintegrated:  $L_{max} = 81L_{min}$ .

A representative series of earthquakes in a given region can be obtained only if the size of the massif being deformed at a constant velocity is about two orders of magnitude greater than the volume of source of the strongest earthquake possible. Otherwise, the regional earthquake recurrence will be distorted. Let us leave the solution of methodical problems to the experts.

Let us now compare the types of seismicity with different velocities of deformation of the Earth's crust. Following are the characteristic parameters and relationships thereof in a seismic zone:

$V_0$ , the lithospheric volume in which the deformation velocity is constant and a steady-state seismicity has established itself;

$\dot{\epsilon}$ , shear deformation velocity;

$\Delta\sigma^*$ , stress limit at inhomogeneities, which causes fractures to appear;

$l_0 = \frac{v \Delta\sigma^*}{\dot{\epsilon} \mu}$ , ( $\mu = \rho c_t^2$ ), size of the inhomogeneities involved in the development of disintegration structure;

$L_0 = l_0 \left( \frac{\pi Q}{2} \right)^{1/3}$ , average distance between fractures;

$E_0 = \frac{(\Delta\sigma^*)^2 l_0^3}{2\mu}$ , energy emitted, as a  $l_0^3$ -volume block is separated; one-half this energy is emitted as elastic waves and is considered the earthquake energy;

$N_0 = \left( \frac{L_{max}}{L_0} \right)^3 = 81^3 \approx 5 \cdot 10^5$  1/yr, if  $N_{max} = 1$  1/yr.

For definiteness's sake, let us write the mechanical characteristics of the lithospheric material in the seismic zone:

$\mu = 5 \cdot 10^{11}$  dyne/cm<sup>2</sup>;  $Q = 100$ ;  $\Delta\sigma^* / \mu = 2 \cdot 10^{-4}$ .

The seismically active layer is taken to have the thickness  $H = 100$ km, if  $(V_0)^{1/3} > 100$  km.

The size of inhomogeneity  $l_0 = \frac{10^{-4}}{\dot{\epsilon}}$ ; if  $\dot{\epsilon}$  is measured in 1/yr, this gives  $l_0$  in meters. The size of source of the minimum earthquake

$$L_0 = \left( \frac{\pi Q}{2} \right)^{1/3} \cdot l_0 \approx 5l_0.$$

The class  $K$  of an earthquake equals the decimal logarithm of the energy emitted by elastic waves and measured in joules.

Designating the class of the minimal earthquake as  $K_0$  gives ,

$$K_0 = \lg \left( \frac{5 \cdot 10^{-10}}{\dot{\epsilon}^3} \right); \quad K_i = K_0 + 3 \lg \frac{L_i}{L_0}$$

$$K_{max} \approx K_0 + 6.$$

The volume required for the entire spectrum of earthquakes from  $K_0$  to  $K_{max}$  to take place is

$$V_0 \approx \frac{5 \cdot 10^{-12}}{\dot{\epsilon}^3}; \quad [\dot{\epsilon}] = 1/\text{yr}; \quad [V_0] = \text{km}^3.$$

The area of the seismic zone is

$$S_0 = \frac{V_0}{100} \text{ for } V_0 > H^3.$$

The period of increase of stresses at inhomogeneities  $\Theta = \frac{2l_0}{v}$  defines the preparatory period preceding the earthquake; this is the same for all the classes of earthquakes in a given seismic zone (with the same deformation velocity):

$$\Theta = \frac{3 \cdot 10^{-4}}{\dot{\epsilon}}.$$

The frequency of recurrence of  $K_i$ -class earthquakes equals the number of blocks  $N_i$  of corresponding size  $L_i$  related to the time  $\Theta$ .

The table lists seismicity parameters for three values of the deformation velocity  $\dot{\epsilon} = 10^{-5}; 10^{-6}; 10^{-7}$ , which cover virtually the whole range of deformation velocities in the seismic zone.

**Table 1**

$\dot{\epsilon}$	1/yr	$10^{-5}$	$10^{-6}$	$10^{-7}$
$L_0$ ,	m	50	500	5000
$K_0$		5, 7	8, 7	11, 7
$L_{max}$ ,	km	4	40	400
$K_{max}$		11, 7	14, 7	17, 7
$V_0$ ,	$\text{km}^3$	$5 \cdot 10^3$	$5 \cdot 10^6$	$5 \cdot 10^9$
$S_0$ ,	$\text{km}^2$	$3 \cdot 10^2$	$5 \cdot 10^4$	$5 \cdot 10^7$
$\Theta$	yr	30	300	3000
$N_0$ ,	1/yr	$15 \cdot 10^3$	$1,5 \cdot 10^3$	$0,15 \cdot 10^3$
$N_{max}$ ,	1/yr	$3 \cdot 10^{-2}$	$3 \cdot 10^{-3}$	$3 \cdot 10^{-4}$

*Note. At small deformation velocities, the maximum earthquake source encompasses the whole thickness of the lithosphere. Such catastrophic earthquakes recur once every 1000 yr with an equal probability over an area of 7000 km × 7000 km. An estimate of the probability of such an event over an area of 106 km<sup>2</sup> will prove to be 50 times smaller.*

Effectively, this means that seismicity with a deformation velocity of  $10^{-7}$  1/yr will be deemed irregular, and individual earthquakes will be considered unpredictable, random events.

At deformation velocities  $\dot{\epsilon} > 10^{-5}$ , seismic areas can be less than 100 km in size, and the earthquake preparation period will not exceed ten years. For such local seismic areas, owing to the high frequency of events, various empirical prognostic guides can be elaborated, but these are of little importance, as earthquakes in these areas are of a low energy class and are not very dangerous.

The most common earthquakes worldwide, making the most damage, result from deformation of significant lithospheric volumes at velocities of  $10^{-5}$ – $10^{-6}$  1/yr.

Thus, earthquakes where  $K \approx 12$  over a seismic area of  $10^5$  km<sup>2</sup> will recur 50 times at  $\dot{\epsilon} = 10^{-5}$  1/yr and only 1.5 times at  $\dot{\epsilon} = 10^{-6}$  1/yr; however, earthquakes with a greater energy  $K \geq 13$  will take place only at deformation velocities of  $10^{-6}$  1/yr, and their preparatory period will exceed 100 years.

If within a large area with a small deformation velocity a minor volume arises that is being deformed at a greater velocity, the situation, of course, will complicate. Furthermore, based on the data on earthquake recurrence without due regard for spatial distribution, it will be difficult to establish the maximum earthquake magnitude for the area.

Considering concepts of the structure of disintegration of solid body in close conjunction with those of seismicity makes it possible to quantify the dimensions of seismic areas and focal zones and to refine the very formulation of the problem of the strongest earthquake possible for a given region. The data on the frequency of weak earthquakes are shown to be highly significant in characterizing the seismicity.

### ***3. Criteria of Slow Motion and Disintegration of the Medium in the Earth's Interior***

Rock masses are set in motion by the action of gravity forces on density-variable bodies in the crust and mantle and by the action of tidal waves; the latter provide the energy for deformations, mainly in the lithosphere, by slowing down the Earth's rotation about its axis.

This motion gives rise to tectonic stresses, to the rugged topography of the Earth's surface, and to disintegration-discontinuities in the Earth's crust. The block structure enhances the mobility of the Earth's solid shells because deformation of the blocks is supplemented by their displacements relative to each other.

Any displacements of a finite body in a medium bring about shear deformation along its boundaries. This deformation can either take the form of a creep, or be localized in a narrow extended area with a strongly disturbed continuity (fault).

Disintegration of a continuous medium by propagation of fractures in a solid body results from the presence of scale-variable inhomogeneities of distinctive origins, which furnish stress concentrators and make for initiation of fractures, as the body is deformed.

Let us estimate the parameters of a slow motion in the Earth's interior, during which disintegration takes place and a fault is formed. We can resort to the case discussed in the previous chapter. Suppose there is a flat layer with a thickness  $L$  whose boundaries are being displaced with respect to each other at a velocity  $V$ . In the case of a homogeneous deformation, its velocity inside the layer will be  $\dot{\epsilon} = V/L$ , and stresses at inhomogeneities will be  $\sigma_l = \rho c_t^2 \frac{Vl}{Lv}$ .

The latter formula can be rewritten in the following form:

$$\frac{\sigma_l}{\rho c_t^2} = \frac{V}{v} \cdot \frac{l}{L}.$$

It is readily apparent that the structure of the stressed inhomogeneities in bodies of variable scale  $L$  will be similar if equal stresses occur at inhomogeneities whose sizes are directly proportional to the size of the body,  $l/L = const$ . In this case, the velocity of displacement  $V$  must be the same, irrespective of the body's size  $L$ .

Embryonic fractures can appear at those inhomogeneities where  $\sigma_l/\rho c_t^2$  is as high as the ultimate strength of the material  $\sigma^*/\rho c_t^2$ .

Let us designate the size of an inhomogeneity as such  $l_0$ . The distance  $L_0$  between inhomogeneities of a size  $l_0$  is defined by their concentration in the volume:

$$\left(\frac{L_0}{l_0}\right)^3 = \frac{\pi \cdot Q}{2}.$$

Suppose the disintegration of the body takes place if its thickness  $L$  accommodates ten fractures, i.e.,  $L/L_0 = 10$ . Substituting the adopted relationships in the formula gives

$$\frac{\sigma^*}{\rho c_t^2} = \frac{V^*}{v} \left(\frac{2}{\pi \cdot Q}\right)^{1/3} \cdot \frac{1}{10}.$$

The strength of rocks  $\sigma^*$  has a broad range, but the relationship  $\sigma^*/\rho c_t^2$  is virtually constant. The merit of material in the Earth's lithosphere is on the order of  $10^2$ . The formula contains no other constants; hence, the velocity of displacement of the boundary interfaces  $V^*$  can serve as a convenient criterion of rock disintegration in the massif. Transforming the relationship obtained and substituting  $\sigma^*/\rho c_t^2 = 2 \cdot 10^{-3}$  è  $Q = 100$  gives an estimate for  $V^*$ :

$$V^* = 10v \frac{\sigma^*}{\rho c_t^2} \left(\frac{\pi \cdot Q}{2}\right)^{1/3} \approx 6 \text{ cm/yr.}$$

Departing from this estimate, it can be shown that rock disintegration will occur, where the velocity of displacement of the interfaces bounding the body attains several centimeters per year. At slower velocities, the deformation will advance in the creep mode.

Average shear stresses  $\tau$  at the boundaries of the body can be defined for the ultimate velocity of their displacement,  $V=V^*$ :

$$\tau = \rho c_t^2 \frac{\dot{\epsilon}}{\nu} \frac{2}{\pi \cdot Q} \int_0^{l_0} l dl \ln l .$$

(In the case of creep, the upper integration limit  $l_m = L \left( \frac{2}{\pi \cdot Q} \right)^{1/3}$ , rather than  $l_0$ .)

$$\tau = \rho c_t^2 \frac{2}{\pi \cdot Q} \frac{\dot{\epsilon} l_0}{\nu} = \rho c_t^2 \frac{2}{\pi \cdot Q} \frac{\dot{\epsilon} L}{\nu} \frac{l_0}{L} .$$

Substituting  $\frac{l_0}{L} = \left( \frac{2}{\pi \cdot Q} \right)^{1/3}$  and  $\frac{L}{L} = \frac{1}{10}$ ;  $\dot{\epsilon} L = V^*$ , yields

$$\tau = \frac{\rho c_t^2}{10} \left( \frac{2}{\pi \cdot Q} \right)^{4/3} \cdot \frac{V^*}{\nu} .$$

If  $\rho c_t^2 = 5 \cdot 10^5 \text{ kg/cm}^2$ ;  $Q = 10^2$ ;  $V^*/\nu = 1/10$ , then  $\tau = 6 \text{ kg/cm}^2$ .

It is worthwhile noting the very small shear stress at the boundary of the layer. Such a small resistance of the solid body to a change in shape at slow deformation velocities renders this body similar to a viscous fluid, especially as (in a viscous fluid layer) stresses at the layer boundaries are also proportional to the velocity of displacement of the boundary:  $\tau = \eta \frac{V}{L}$ , where  $\eta$  is the dynamic viscosity coefficient. Going to dimensionless parameters, in order to take a grasp of the similitude of scale-variable flows, gives

$$\frac{\tau}{\rho V^2} = \zeta(Re), \text{ where } Re = \frac{\rho L V}{\eta} \text{ is the Reynolds number.}$$

For laminar flow,  $\zeta(Re) = 1/Re$ ; for turbulent flow,  $\zeta = \text{const}$ . The criterion of transition from laminar to turbulent flow is the experimentally determined critical number  $Re^*$ .

In a solid body, when the velocity of displacement exceeds the critical value, strongly stressed inhomogeneities appear, at which fractures originate. Disintegration also results in the deformations in the body ceasing to be homogeneous.

Recall that, for all the similarity of these phenomena, they differ significantly in that, in a fluid, structure arises due to kinetic energy and, in a solid, to elastic energy. The mechanisms of energy dissipation are also different.

In conclusion, note one more feature of solid-body disintegration in the course of a slow deformation—that it tends to localize in a narrow zone. In light of the above concepts, this effect should be explained by mere kinematic considerations rather than by any energy advantage: the thinner the layer, the sooner disintegration is completed, because a thin layer has a higher velocity of deformation  $\dot{\epsilon} = V/L$ .

#### 4. On the Landslide Regime of Slopes

Mass transport down slopes by gravity is one of the prominent topography-forming processes.

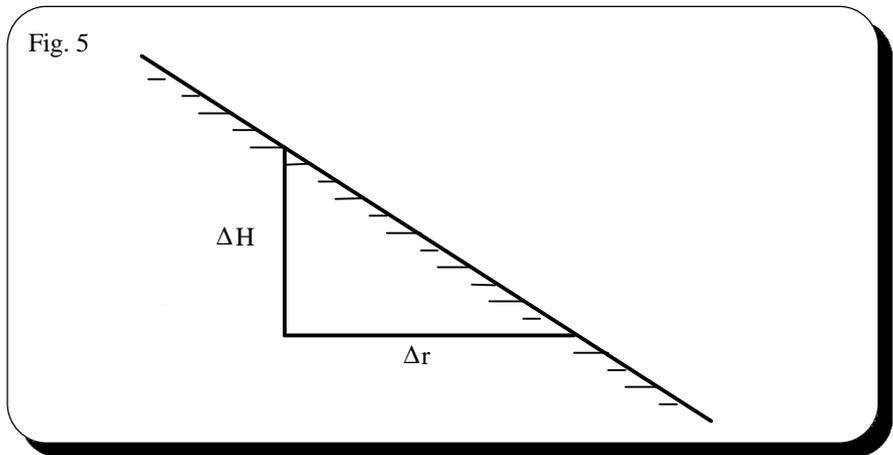
Exogenous processes are highly diversified. The greatest hazard is presented by mud torrents, avalanches, and landslides. Mud torrents and avalanches arise where loose clastic material can accumulate on the upper levels of slopes and valleys. When triggered by earthquakes or when strongly moistened, the loose clastic pile loses stability and, moving downhill, increases in mass. A mud torrent or an avalanche thus builds up its destructive energy.

With landslides, the situation is quite different. Their preparation is concealed and is caused by deformation of the rock massif under the slope. Some slopes preserve evidence of multiple landslides, suggesting that this process is recurrent. Although each particular event can be triggered by an earthquake or by a change in the water regime, these cannot be deemed the reasons for landslide generation.

Preparation of a landslide begins, as an interface forms that separates a certain volume of rock mass from the massif, and a shear strain becomes localized along this interface. Along the interface, rock porosity and permeability increase. Provided this zone gets saturated with water, the shear resistance can become nearly zero, which results in a landslide.

The deformation eventually resulting in the formation of the landslide body advances very slowly and is accompanied with disintegration of the continuous medium.

The role of the geometry factor in the landslide formation is defined by the condition of equilibrium of the forces acting on the block being detached from the massif (Fig. 5).



The forces acting along boundaries  $\Delta H$  and  $\Delta r$  in length are, respectively,  $\sigma\Delta H$  and  $\sigma\Delta r$ . Their projection on the fall line, down which the rock mass moves, must equal the projection of the gravity force:

$$-\sigma \cdot \sqrt{\Delta H^2 + \Delta r^2} = \frac{\Delta r \cdot \Delta H}{2} \rho g \frac{\Delta H}{\sqrt{\Delta H^2 + \Delta r^2}},$$

where  $\rho$  - is rock-mass density, and  $g$  is gravitational acceleration.

After certain transformations, setting  $|\Delta H/\Delta r| < 0,5$ , and ignoring  $(\Delta H/\Delta r)^2$  in comparison to unity, the linear size of the block  $L = \Delta H$  is defined as

$$L \approx \Delta H = \frac{2\sigma}{\rho g} \cdot \frac{1}{|\Delta H/\Delta r|}$$

The size of the landslide block is a function of the slope gradient and shear stress in the  $L$  layer. The danger of separation of the block arises when the velocity of downhill motion attains  $V = 6$  cm/yr. This velocity corresponds to a shear stress

$$\sigma = \rho c_t^2 \left( \frac{2}{\pi \cdot Q} \right)^{4/3} \cdot \frac{V^*}{V} \cdot \frac{1}{10}.$$

Substituting  $\rho c_t^2 = 10^5 \text{ kg/cm}^2$ ;  $\pi Q/2 = 100$ , yields

$\sigma = 10^5 \cdot 0,2 \cdot 10^{-2} \cdot 10^{-1} \cdot 10^{-1} = 2 \text{ kg/cm}^2$ . This value of shear stress makes it possible to define the size of the landslide block ( $\rho = 2 \text{ g/cm}^3$ ):

$$L = 20 \frac{1}{|\Delta H/\Delta r|} \text{ m}.$$

The preparatory period of the landslide event is determined by the period of rock disintegration inside the block. The localization of shear strain along the block boundaries takes a comparatively short time because the deformation zone becomes smaller, and the deformation velocity increases accordingly. The disintegration process in a volume usually terminates in the formation of a block structure, while the average deformation measures a few percent.

Therefore, in terms of the order of magnitude, the preparatory period of a landslide can be estimated by the formula

$$\Theta = \frac{L}{V^*} \varepsilon^*.$$

Assuming that the general deformation prerequisite for disintegration inside the future block to be completed is  $\varepsilon^* = 0,01$  and that  $\Delta H/\Delta r = 0,5$  gives

$$\Theta = \frac{40}{0,06} \cdot 0,01 \approx 6 \text{ yr}.$$

With decreasing slope gradient, the preparatory period increases, and so does the block size.

In a landslide, the block mass is displaced virtually instantaneously over a distance equal to the size of the block. Relating this distance to the landslide preparation period yields the effective velocity of mass transport by landslides

$$V_{ef} = \frac{L}{\Theta} = \frac{V^*}{\varepsilon^*} = 6 \text{ cm/yr} \cdot 100 = 6 \text{ m/yr}.$$

As appears from this case, the effective mass-transport velocity in the  $L$ -thick layer can attain a significant value.

The above scheme of landslide preparation can provide the basis for early predictions of the landslide hazard.

## 5. Horizontal Discontinuities in the Earth's Crust

A motion that could give rise to horizontal layers of disintegrated rock must be directed virtually along the equipotential surface. This means that the proper gravity field cannot provide the energy for such a motion. Of course, one can maneuver such a configuration of the slide surface at the base of the plate being deformed as to result in a horizontal motion forced by gravity, but this would not be typical of the terrestrial environment.

At the same time, it is rather intriguing that the effective thickness of the atmosphere (~10 km), the ocean depth, and the Earth's crust thickness are quite commensurable. It is also surprising that all these are close to the characteristic linear value obtained by dividing the square of the linear velocity of the Earth's rotation about its axis by the gravitational acceleration:

$$\frac{V^2}{g} = \frac{(350 \text{ m/s})^2}{10 \text{ m/s}^2} \approx 10^4 \text{ m} = 10 \text{ km}$$

This coincidence makes one wonder about the role of mechanical motion in the formation of the fine structure of the geospheres.

Among the conceivable explanations, the most realistic is that this structural organization is related to the action of tidal waves. The propagation velocity of gravitational waves in a thin layer of heavy fluid is  $c = (gH)^{1/2}$ ; consequently, in the atmosphere and in the ocean, these will move virtually at the rotational velocity  $(gH)^{1/2} = (10 \cdot 10^4)^{1/2} \approx 330 \text{ m/s}$ . Having the same velocity as the tidal wave, the gravitational wave, as it moves, will be replenished with energy by the action of tidal forces.

Evidently, this kind of "resonance" regime can provide the maximal mechanical energy for the structural arrangement of the geospheres. Any quantitative estimates to confirm this assumption require a special study.

Let us resume the consideration of the structural organization of the Earth's crust. The magnitude of displacement on the surface of the solid Earth's shell at the crest of the tidal wave equals about 0.5 m. Compared to the elastic-wave velocity, the tidal wave moves slowly; hence, for the Earth as a whole the tidal deformation can be estimated by a quasisteady-state approximation. In this situation, however, in the upper lithospheric layers, displacements can arise that will contribute to the tidal motion just as it happens in the ocean.

The block structure of the Earth's crust precludes its identification with an elastic shell. To clarify the kinematics of surface layers in the tidal wave, a heavy-fluid model is best suited. Gravitational wave in a deep enough fluid layer propagates at a velocity that depends on the wavelength. The amplitude of oscillations decreases rapidly with depth. The velocities of particle displacement in the gravitational wave are described by the following formulas:

$$v_x = A\omega e^{-kz} \sin(kx - \omega t),$$

$$v_z = A\omega e^{-kz} \cos(kx - \omega t).$$

The motion along the surface is directed along the  $x$  axis; the  $z$  axis points from the surface into the depth of the fluid.

$\omega = \sqrt{k \cdot g}$  is the frequency of oscillations;  $\lambda = \frac{2\pi}{k}$  is the wavelength, and

$A$  is the amplitude.

The velocity of propagation of the gravitational wave is

$$c = \frac{1}{2} \sqrt{\frac{g}{k}} = \sqrt{\frac{g \cdot \lambda}{8\pi}}.$$

Of interest are only those waves that can receive mechanical energy, as they propagate, owing to the fact that their velocity coincides with that of the tidal wave. Under the mid-latitudes, the tidal wave propagates at  $\sim 350$  m/s. Substituting this value into the formulas yields

$$1/k = 48\text{km}; \lambda = 300\text{km}.$$

In the context of the kinematic system proposed, it is easy to estimate the depth at which the disintegration of the solid medium can run high.

In one cycle, the running surface waves irreversibly displace particles in the direction of water motion over a distance of about  $A/Q$ , where  $Q$  is the merit of the solid-the Earth's crust.

As the amplitude decreases with increasing depth, the upper layers are displaced slowly relative to the lower. This typical near-surface shear deformation can be taken as a derivative with respect to  $z$ :

$$\varepsilon = \frac{1}{Q} \cdot \left. \frac{d(Ae^{-kz})}{dz} \right|_{z=0} = \frac{A \cdot k}{Q}.$$

In one year, 730 cycles take place, hence the deformation velocity is

$$\dot{\varepsilon} = 730 \frac{A \cdot k}{Q} \text{ 1/yr.}$$

The velocity of displacement of the surface relative to a layer at the depth  $H$  increases with depth:

$$V = \dot{\varepsilon} H = 730 \frac{A \cdot k}{Q} H.$$

A discontinuity can appear in the  $H$  layer if the surface is displaced at a relative velocity  $V^* = 6$  cm/yr. This yields the minimum depth at which the Earth's crust can suffer disintegration through the action of the tidal forces

$$H = V^* \frac{Q}{A \cdot k \cdot 730}.$$

Assuming  $V^* = 6$  cm/yr,  $Q = 100$ ,  $A = 5$  cm, and  $1/k = 48$  km gives  $H \approx 8$  km.

If the contribution from the gravitational wave into the tidal motion is less than 10% (5 cm), the depth of the disintegration layer will increase. Unfortunately, to our knowledge, no attempts have been made to discern the gravitational wave proper on the records of the tidal wave.

## Chapter IV. Differentiation of Matter in the Earth

The presence of the heavy core testifies to a differentiation of matter having occurred as early as the planet Earth was formed, and its mass was almost as great as today. Having arisen through coalescence of bodies, the primordial Earth naturally had a heterogeneous composition and, as a result, carried in its body some density inhomogeneities of various scale. In the proper gravity field, these inhomogeneities generated an enormous amount of potential energy. The differentiation of matter and related motion and heating of the solid medium change the Earth's internal structure, forming steady structures and energy flows. Estimating the role of the differentiation of matter in the modern epoch and the form it takes presents a certain interest. As we are dealing with the stored mechanical energy realized through differentiation of matter, it is reasonable to first study a mechanical model of this process.

I propose a very simple model describing the motion of density inhomogeneities in a solid medium in the Earth's gravity field. This model makes it possible to quantify, in a very general form, the parameters of mechanical motion and related thermal effects.

(1) The mechanical model of compositional differentiation is based on the concept of density heterogeneity of the primordial Earth. No experimental data are available to properly substantiate this model. Hence, we will make use of the heterogeneous structure that arises through a random coalescence of heavy and light bodies of the same size (an approach suggested by K.E. Gubkin). The method used in constructing this structure is presented in the Appendix. Noteworthy is the qualitative feature of structures of this kind. Consider a medium composed of light and heavy cubes of equal size in such a manner that, if scanned, the probability of encountering a cube of each particular sort equals 1/2. With decreasing resolution of the scanning device, along with volumes of a roughly average density, a steady proportion of volumes with elevated and reduced densities will be encountered. If the resolution is defined by the size of the scanning spot measured in the lengths of the edges of the basic cube, the density difference between heavy and light volumes and the average will be inversely proportional to the resolution. In other words, increasing the volume over which the density is averaged results in the density difference between randomly chosen equal volumes decreasing in reverse proportionality to their linear dimension. The frequency of encountering neutral (average-density), heavy, and light volumes in this medium remains the same at all the hierarchical levels.

Let us designate the linear size of the basic cube as  $L_*$ . Suppose a heavy basic cube has a density of  $8 \text{ g/cm}^3$  and a light basic cube,  $2 \text{ g/cm}^3$ . The average density of the medium composed of these cubes will then be  $5 \text{ g/cm}^3$ . In this medium, let us consider volumes of different linear size:  $L_e = 2^k L_*$  ( $K$  is the scale level). At each particular level, one can recognize volumes with a roughly average density and volumes differing in density from the average by  $\pm \Delta \rho_K$ . Apparently, a

conventional boundary between those volumes deemed neutral and those termed heavy or light inhomogeneities must be placed in such a manner that the probability of encountering a neutral volume by random sampling will be 1/2; a heavy volume, 1/4; and a light volume, also 1/4. In this case, it turns out that for  $L_e > 4L$ , the relationship

$$\Delta\rho_K = \pm 4 \frac{L^*}{L_K} \text{ g/cm}^3 \text{ holds true.}$$

The medium constructed by a random packing of equally sized basic cubes (heavy and light with an equal probability) has a hierarchical regular structure of inhomogeneities.

The fraction of space occupied by inhomogeneities at each hierarchical level is the same: 1/4 for light and 1/4 for heavy inhomogeneities.

(2) The presence of potential energy in the light and heavy volumes is a necessary, but not sufficient, condition for a motion to begin. The force resisting motion of a body in a condensed medium is directly proportional to the surface area of the body and to the shear stress along its boundaries.

A slow deformation of a solid body gives rise to a creep, so that shear stresses will be proportional to the velocity of the body,  $V$ :

$$\sigma \sim \rho c^2 \frac{V}{v} \cdot \left( \frac{2}{\pi \cdot Q} \right)^{4/3},$$

where  $\rho c^2 = \mu$  and  $v = 2 \cdot 10^{-6}$  cm/s.

In a viscous fluid, as the velocity of motion exceeds a certain critical value, turbulence arises in the ambient flowing around the body; likewise, as a body moving in a solid medium attains a certain critical velocity  $V^*$ , the medium disintegrates in the immediate vicinity of the body.

This circumstance appears to be very important in discerning those volumes that differ in density from the average and are capable of participating in the mechanical differentiation of matter.

The scale-variable density inhomogeneities we have recognized based on a statistical analysis, moving in the gravity field, will multiply rapidly by exchanging mass with the medium unless they acquire well-defined physical boundaries when in motion.

Disintegration of the medium along the body's boundaries occurs only when the velocity  $V^*$  is attained. Consequently, all the bodies moving at a slower velocity cannot contribute essentially to the differentiation of matter.

The equal velocity of variously sized bodies,  $V = V^*$ , means that shear stresses ( $\sigma$ ) at the boundaries of these bodies will also be equal. Assuming

$\sigma^*/\rho c_t^2 = 2 \cdot 10^{-3}$ ,  $Q = 100$ ,  $L/L_0 = 10$ , and  $\rho c_t^2 = 5 \cdot 10^{12}$  dyne/cm<sup>2</sup> gives

$$V^* = v \frac{\sigma^*}{\rho c_t^2} \frac{L}{L_0} \left( \frac{\pi \cdot Q}{2} \right)^{1/3} \approx 6 \text{ cm/yr,}$$

$$\sigma = \rho c_t^2 \left( \frac{2}{\pi \cdot Q} \right)^{4/3} \cdot \frac{V^* L_0}{v} \sim 10^7 \text{ dyne/cm}^2.$$

By composing an equation of forces acting on the inhomogeneity, it is possible to define the size of the basic cube  $L_*$  that normalizes the hierarchical structure obtained.

Assuming that the inhomogeneities have a cubic shape, we obtain the following expression for the volume gravitational force:

$$\Delta\rho_K \cdot gL_K^3 = \frac{4L_*}{L_K} gL_K^3 = 4L_* gL_K^2.$$

The surface resistance to motion equals the shear stress multiplied by the surface of the body:  $10^7 \cdot 6 \cdot L_\epsilon^2$  dyne.

Equating the volume force and surface force yields

$$4L_* \cdot 10^3 L_\epsilon^2 = 10^7 \cdot 6 L_\epsilon^2.$$

Hence,  $L_* = 1,5 \cdot 10^4 \text{ m} = 150 \text{ m}$ . As follows from the above force balance, for all the inhomogeneities  $L_\epsilon > 4L_*$  ( $\hat{\epsilon} > 2$ ), the size  $L_\epsilon$  on the left and right sides of the equation is reduced. This means that, at all the hierarchical levels, inhomogeneities are equally responsive to the action of the gravity force. A dynamic similitude of moving inhomogeneities takes place ( $\hat{\epsilon} \geq 2$ ), which implies an equal velocity of the motion, equal stresses at the boundaries, and an equal probability of encountering an inhomogeneity at any point, as at each level heavy and light inhomogeneities occupy equal fractions of the space.

It is thus natural to regard  $k = 2$  as the lowermost level in the hierarchy.

$L_2 = 4L_* = 600 \text{ m}$ . The upper level of the hierarchical structure will be addressed later.

(3) All the inhomogeneities at each particular level exist under the same conditions, hence their involvement in the differentiation of matter is equally probable. Yet all of these cannot be in motion simultaneously, as their ability to move depends on their medium: a heavy or light inhomogeneity can move under the action of the Archimedian force only if its immediate neighborhood has an average density.

Continuing to use the cubic shape in quantitative estimates, assume that the neighborhood favorable for an inhomogeneity to realize its potential consists of six (according to the number of faces) neutral cubes of the same size. The probability of such a situation equals the product of probabilities of one heavy (light) and six neutral cubes occurring together:

$$\frac{1}{4} \cdot \left(\frac{1}{2}\right)^6 = \frac{1}{256}$$

Therefore, at any given instant of time, the moving density inhomogeneities at each particular level occupy 1/256 of the space and, because of their scarcity, hardly interact with each other. The number of light buoyant inhomogeneities equals the number of heavy sinking ones, hence it can be inferred that neutral volumes (those with an average density) remain immobile. The fluxes of matter bound to the surface and to the center compensate each other in volume.

(4) The total mass flows in one direction because of the similitude of motions at all the levels equals the lowest level flow multiplied by the number of levels in the hierarchical structure. The excess mass, however, is transported mainly by the lowest level inhomogeneities, while the rest, taken together, transport just as much.

In fact, the mechanical differentiation of matter is performed by density inhomogeneities of the first three or four levels.

Their sizes are as follows: the lowest,  $\hat{e}=2$ ,  $L_2=600\text{m}$ ;  $\hat{e}=3$ ,  $L_3=1200\text{m}$ ;  $\hat{e}=4$ ,  $L_4=2400\text{m}$ ;  $\hat{e}=5$ ,  $L_5=4800\text{m}$ .

Excess mass is transported to the core-mantle interface by two simultaneous flows of heavy and light inhomogeneities. Heavy inhomogeneities occupy the site of light ones; hence, the mass supplied by the sinking inhomogeneities must be doubled.

Let us estimate the excess mass transfer at the lowest level ( $k=2$ ,  $L_2=600\text{ m}$ ). The volume of all the moving inhomogeneities at one level is  $1/256$  of the mantle volume:

$$\sum L_K^3 \approx \frac{10^{27}}{256} \approx 4 \cdot 10^{24} \text{cm}^3.$$

The number of the moving heavy volumes of the lowest level ( $L_2=6 \cdot 10^4 \text{cm}$ ) is

$$N_2 = \frac{\sum L_K^3}{L_2^3} = \frac{10^{27}}{256 \cdot 216 \cdot 10^{12}} = 2 \cdot 10^{10}.$$

All of these are sinking at the velocity  $V^*=6\text{cm/yr}$ . The time interval  $\tau$ , required for these volumes to pass through the mantle thickness of 3000 km equals

$$\tau = \frac{3 \cdot 10^8}{6} = \text{yr}.$$

The mass supplied by the heavy inhomogeneities  $L_2$  equals their volume times  $\Delta\rho=1\text{g/cm}^3$ , and the total mass transported into the core by the flows of heavy and light inhomogeneities of all the levels is four times greater:

$$\Delta m = 4 \cdot \frac{10^{27}}{256} \cdot 1 \approx 1,6 \cdot 10^{25} \text{g}.$$

Relating this mass to time  $\tau$  yields the rate of increase of the core's mass:

$$\frac{1,6 \cdot 10^{25}}{5 \cdot 10^7} \approx 3 \cdot 10^{17} \text{g/yr}.$$

For the mass of the core to attain its present value, the initial mass must have been increased roughly twofold, i.e., by  $10^{27} \text{g}$ .

The mechanical differentiation of matter must take at least  $3 \cdot 10^9$  years to supply this mass to the core.

This estimate shows that the inner geospheres are formed through differentiation of matter in a virtually steady state mode throughout the Earth's history, and the current epoch must not differ in this respect from the previous ones.

The buoyancy of the light volumes and the sinking of the heavy near the lithospheric boundaries can result in the reappearance of instability areas in the Earth's crust that expend their energy for deformational processes.

The interaction between the density inhomogeneities and tidal waves also deserves a special consideration.

(5) Overcoming the resistance of forces that act on their surface, the moving bodies perform a work that results in the heating of the bodies and the medium. If the body is small, the surrounding area it heats up is much greater than the body itself, the body's boundaries become vague, and the intense mass exchange with the medium blurs the body.

The thermal criterion that can be used to estimate the minimum size of a stable body holds that the velocities of the body and of the temperature front must be equal. The velocity of motion  $V^* = 6$  cm/yr. We will define what can be conventionally termed the temperature front using the temperature conductivity coefficient  $K^2$ :

$V_\delta = k^2/L$ , where  $L$  is the size of the moving hot inhomogeneity.

For rocks,  $k^2 \sim 3 \cdot 10^5$  cm/yr. Equating the velocities  $V_\delta$  and  $V^*$  gives

$$L_{\min} = \frac{30 \cdot 10^4}{6} = 5 \cdot 10^4 \text{ cm} = 500 \text{ m.}$$

A body that moves at a velocity of 6 cm/yr will sustain its boundaries and mechanical stability, provided its size exceeds 500 m. The size of inhomogeneities at the lower level of the hierarchical structure to which we refer equals 600 m, thus satisfying this condition.

The larger the size of a moving compositional inhomogeneity the less its difference in density from the medium. Thus, for bodies with a size  $L \sim 100$  km this difference can be ca. 1/1000 of the average density. But density differences of the same order can arise at a temperature difference of a mere 100 K. For this reason, a constraint must be put on the involvement of very large bodies in the mechanical differentiation. Apparently, the upper hierarchical level should be limited by  $k = 6$ ,  $L_6 = 64$  km. Therefore, only five levels,  $2 < k < 6$ , participate actively in the mechanical differentiation of matter.

(6) The sinking and rising bodies of the same size and with equal  $\Delta\rho$  receive, as they move, equal energy, but their heating en route through the whole thickness of the mantle will be different because their starting states are different.

Let us assess the heat flow generated by those bodies that are heated as they rise through the mantle.

A body receives the heat energy at the expense of a change in its gravitational potential:

$$\Delta\rho_K \cdot L_K^3 \cdot gH, \text{ where } H=3000\text{km.}$$

One-half of this energy is spent for heating the medium, while the other half remains in the body. The density difference between the body and the average value decreases with increasing average size of the body. Hence, it suffices to define the energy of the lowest level bodies that contribute over one-half of the total energy supplied to the surface.

The number of lowest level bodies ( $L_2=600\text{m}$  and  $\Delta\rho=1\text{g/cm}^2$ ) in the mantle equals  $2 \cdot 10^{10}$ . In a period of  $5 \cdot 10^7$  yr, all of them will reach the surface, and each will transfer the energy

$(1/2) \cdot \Delta \rho_2 \cdot L_2^3 g H = (1/2) \cdot 216 \cdot 10^{12} \cdot 10^3 \cdot 3 \cdot 10^8 = 3 \cdot 10^{25}$  erg. Hence, the total heat flow through the surface will be

$$\frac{2 \cdot 3 \cdot 10^{25} \cdot 2 \cdot 10^{10}}{5 \cdot 10^7} \approx 2 \cdot 10^{28} \text{ erg/yr.}$$

Nearly the same amount of energy must be supplied by heavy bodies to the core, resulting in its heating. This process, however, must be limited in time, as the heat capacity of the core is limited. The energy density in lowest level bodies ( $L_2=600\text{m}$ ) on their passing through the mantle is

$$\frac{3 \cdot 10^{25}}{216 \cdot 10^{12}} \approx 1,5 \cdot 10^{11} \text{ erg/cm}^3.$$

Evidently, as the inner energy of the core approaches this value, the growth of the core's temperature will slow down, and various heat-removing mechanisms will come into play.

The characteristic time of intense energy pumping into the core  $10^{26} \text{ cm}^3$  in volume is

$$\frac{1,5 \cdot 10^{11} \cdot 10^{26}}{2 \cdot 10^{28}} \approx 10^9 \text{ yr.}$$

Considering the heat removal, this time must be essentially greater because the temperature growth is likely to slow down exponentially, with the same characteristic time of  $10^9$  years.

As half the energy is dissipated by bodies moving in the mantle, the latter is uniformly heated throughout. The energy absorbed by the mantle due to the motion of heavy and light bodies equals  $4 \cdot 10^{28}$  erg/yr.

Supposing this energy is distributed evenly through the Earth's volume and setting the heat capacity at  $4 \cdot 10^7 \text{ erg/K} \cdot \text{cm}^3$  gives the annual temperature increment

$$\frac{4 \cdot 10^{28}}{4 \cdot 10^7 \cdot 10^{27}} = 10^{-6} \text{ K/yr.}$$

This annual increment of the average temperature testifies to an essential energy contribution from the mechanical differentiation of matter into the Earth's thermal processes.

(7) With progressive heating of the Earth, large moving bodies become increasingly important in the heat transfer. Bodies of the lowest hierarchical level act as generators and sources of heat. Major, high-level bodies receive little heat through motion, but instead they transport their inner energy almost intact from one part of the space to another. Thus, a large body, having sunk to a great depth, turns out to be cooler than the surrounding ambient and, hence, robs it of some energy. Conversely, an ascending large body, retaining its initial internal energy, winds up overheated so strongly as to melt on decompression. Supposedly, at a temperature difference of ca. 1000 K near the core boundary and close to the lithosphere, heat flows related to large moving bodies and equalizing the mantle temperature will be commensurable with the heat produced by the moving bodies of the lowest hierarchical level. It is significant that compositional differentiation is accompanied by the reappearance of nonsteady-state temperature fields.

As for the distribution of heat sources in the mantle, the greatest amount of heat, in agreement with the model for mechanical differentiation of matter, must be produced in the upper mantle near the lithospheric boundary.

The model of mechanical differentiation of matter demonstrates how a medium composed of heterogeneous bodies placed randomly in the space develops, in the gravity field, a hierarchical structure of bodies whose density varies systematically from one level to another. This structure controls the differentiation of matter. Its stability is based on the self-maintaining hierarchy, which in turn provides the basis for steady-state regimes.

## Appendix

### **Hierarchical Structure Arising when Black and White Cubes of the Same Size Are Packed Randomly with Equal Probability of Choice of Color**

By structure, we will mean the distribution of dark- and light-colored volumes against the general gray background at different scale levels.

Suppose we have at our disposal two bags: one contains white cubes ( $a_0$ ) and the other, black cubes ( $b_0$ ). We will randomly choose cubes each time by tossing a coin and will pack them closely in a large box. In the end, we will find that cubes of different colors are intermixed so that large volumes in the box look equally gray. However, if we learn to distinguish semitones, the color uniformity will be put to question.

Let us consider the variants of packing of black and white cubes in a cubic volume that accommodates eight cubes. Suppose at the beginning it is filled only with white cubes. One white cube can be replaced with black in eight possible ways, and two cubes, in 24 ways, etc. These numbers are listed in the table.

**Table 2**

Number of possible variants of distribution of black and white cubes in a volume containing eight cubes

Combination of (a) white and (b) black	$8a_0$	$7a_0$	$6a_0$	$5a_0$	$4a_0$	$3a_0$	$2a_0$	$1a_0$	$8b_0$
		$1b_0$	$2b_0$	$3b_0$	$4b_0$	$5b_0$	$6b_0$	$7b_0$	
Number of variants	1	8	24	56	78	56	24	8	1

The total number of variants is  $2^8 = 256$ . If we single out purely white and purely black combinations, we will find one of each in a box that accommodates  $8 \cdot 256 \sim 2 \cdot 10^3$  cubes. As the volume increases, the probability that it consists of cubes of the same color becomes infinitesimal.

Let us try and introduce another kind of ranking of volumes by color. Suppose a volume of eight cubes is gray if the number of cubes of one color in it exceeds that

of the other color by a factor of two or less; accordingly, the remaining volumes will be classed as light (a) and dark (b). In this case, the probability of finding in our box dark or light volumes will be  $\sim 1/7$ , and the probability of finding gray (c) will be  $5/7$ . Exactly these volume fractions of the box will be occupied by cubes of each color.

If we study in the same manner this new set of cubes ( $L_1=2L_0$ ) regarding the variants of distribution of dark, light, and gray cubes in a cubic volume with a linear size of  $L_2=2L_1=4L_0$ , the following result will be obtained: the probability of a combination yielding a neutral (gray) color equals  $1/2$ , and that of a dark or light color equals  $1/4$ . In this case, the  $L_2^3$  light and dark volumes are faded as compared to the  $L_1^3$  volumes by a factor of ca. two: the number of excess white and black cubes in a unit of volume giving it a contrasting shading against the gray environment has halved.

Repeating this procedure, we find that those fractions of the general volume, occupied at the next level ( $L_3=8L_0$ ) by light, dark, and neutral cubes, remain unchanged- $1/4$ ,  $1/4$ , and  $1/2$ . At the same time, the number of excess black and white cubes in a unit of volume of shaded cubes decreases with their size, following the law

$$n = \frac{1}{3} \cdot \frac{L_2}{L_k}, \text{ where } L_k = 2^k L_0.$$

If an imaginary medium composed of black and white cubes of the same size  $L_*$  is viewed from a varying distance-so that, as the spatial resolution decreases, the sensitivity to semitones accordingly increases-whatever the size of the portion of space being viewed, we will face the same variegated pattern: a gray background randomly splashed with dark and light spots. With varying scale, the hierarchical structure of the space remains similar to itself.

It is clear that in order to see this effect it is necessary to enhance the sensitivity to semitones, as the field of vision expands. In nature, though, this phenomenon is quite common.

Thus, the effect of gravitational forces on a medium with density inhomogeneities grows with their size, so that even a minor density difference, when found in a great volume, becomes significant for mechanics.

## **Chapter V. The Earth's Hydrodynamic Plumbing System**

The hydrosphere is commonly identified with the world ocean. Hydromechanics of the oceans is quite an independent division of geomechanics. On the surface of continents, rivers transport significant masses of water-supported solid particles derived from eroding hard rock, thus participating in the landscape-forming processes.

Free water also occurs in the Earth's interior, at least in the crust. Water fills in porous, permeable strata and voids and fissures in rock massifs.

The continuous motion of masses in the Earth's interior, reproducing dynamic structures, forms layers of disintegrated rocks permeable for fluids. As a result, inside the Earth there arises a plumbing system for circulating waters to maintain the exchange of matter between the lithosphere, on the one hand, and the hydrosphere and atmosphere, on the other.

The influence of water on mechanical properties of disintegrated solid medium will be demonstrated on the example of sand. Dry sand consists of randomly packed grains; compressive force at their contacts is defined by the weight of sand above a given horizon divided by the number of contacts inside this horizon. Cohesive forces between sand grains are absent. The force required for one layer to be shifted along the other (friction force) is directly proportional to the compression force at the contacts. The internal friction also determines the angle of rest of the sand heap.

When sand is moistened, the capillary forces bind sand grains together by a water film. Such sand can sustain a minor vertical escarpment.

If all pores in sand are filled with water, its behavior will depend on the manner in which the load is distributed between the water and sand-grain contacts. If the entire load is borne by the sand framework, the surface topography with height differences will be stable, as can be seen on a river bed.

If again, for some reason, some sand grains lose contact with each other, the load will be borne by the water, hence the friction force will be so small as to result in the so-called liquefaction of the sand ground: the angle of rest becomes zero, and the water-saturated sand mass will spread like water. Sand grains moving freely in the liquefied sand tend to pack more closely and squeeze water out of the pores, eventually causing the arrest of sand motion. The sand liquefaction effect can be caused by dynamic displacements along the boundaries or inside the sand volume. A porous body can become water-saturated provided there are conduits between pores.

Fluid flowing through the body can increase or reduce its permeability. This depends on whether fine particles are leached from or deposited in the porous body, which in turn depends on the velocity of fluid flow and the size of pores and particles. There are also some other mechanisms that control permeability and depend on chemical interaction between water solutions and solid particles of diverse chemical composition.

In permeable solid bodies, fluid moves by action of pressure gradient and by gravity. Porous permeable beds arise in zones of shear deformation, such as block boundaries. Block motions define shear stresses at the block boundaries.

As for the pressure in the porous space between the blocks, this is determined by volume changes, as the blocks converge or part. Because the average density remains constant, these block displacements have a quasiperiodical nature. The jostling blocks act as pumps that keep the hydrodynamic plumbing system running.

According to B.N. Golubov, the phenomenon of the Caspian Sea is based on water exchange with the underground hydraulic plumbing system. For over the next ~20 years after 1920, the level of the Caspian Sea dropped rapidly by ca. 2 m, and 1000 km<sup>3</sup> water was lost. In the 35 years to follow, the sea level dropped another 1 m or so. Since 1977, the sea level has been rising at a rate of ~0.1 m/yr.

Picture a scheme of water exchange between the Caspian and deep-seated permeable beds that models this phenomenon in general features.

Suppose the porous bed occurs at a depth greater than 5 km and has a porosity of 0.1 and permeability of 10<sup>-11</sup> m<sup>2</sup>. The area of the bed actively involved in water exchange with the Caspian is ~10<sup>6</sup> km<sup>2</sup>.

Assume that the hydrodynamic connection between the underground reservoir and the Caspian is weak, and water pressure in this reservoir can differ notably from the hydrostatic pressure.

The shoaling of the Caspian in the 1920s and 1940s may have occurred because of the escape of water into this underground reservoir. Let us estimate the volume of the porous space in this reservoir capable of accommodating 1000 km<sup>3</sup> of water. Suppose the initial water pressure in it was roughly equal to the atmospheric pressure at that time. As the pressure becomes as high as the hydrostatic pressure at the 5-km depth (500 kg/cm<sup>2</sup>), the compressibility of water will result in volume  $V$  accommodating an extra 2.5%  $V$ . Hence, the pore volume required to contain 1000 km<sup>3</sup> water will be

$$V = \frac{1000}{0.025} = 4 \cdot 10^4 \text{ km}^3.$$

At a porosity of 0.1, the volume of the porous layer will be 4·10<sup>5</sup> km<sup>3</sup>. Dividing the volume of the layer by the area gives the average thickness of the layer:

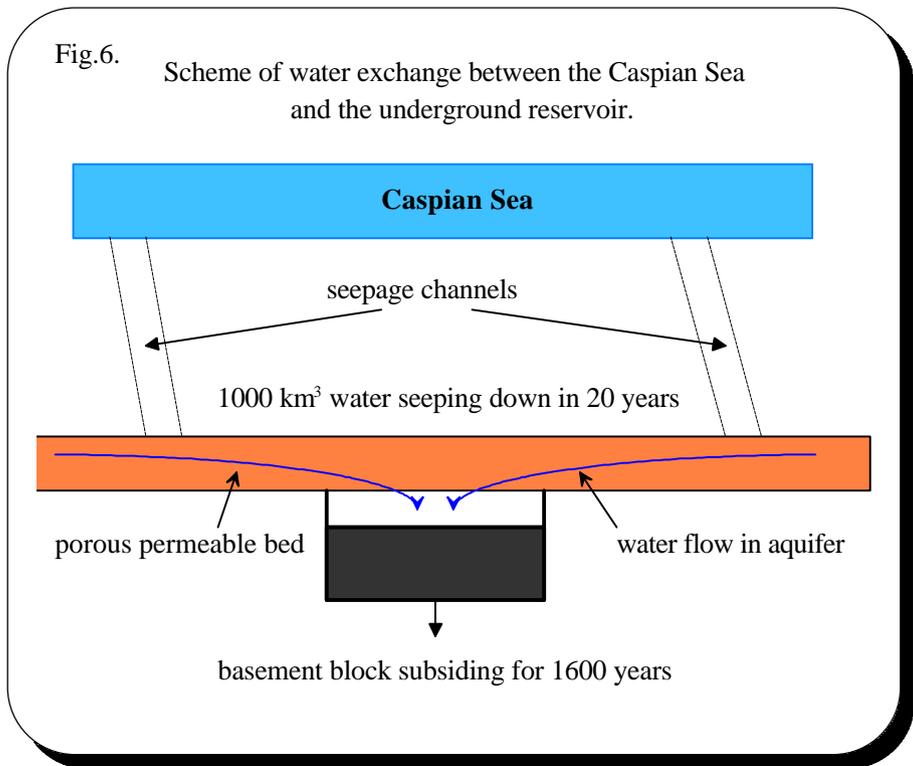
$$h = \frac{4 \cdot 10^5}{10^6} = 0,4 \text{ km}.$$

A pressure reduction to the atmospheric value in the pores of the permeable layer may have been caused by subsidence of one or several blocks of the crystalline basement.

Supposing that water seepage from the Caspian Sea into the underground reservoir prior to 1920 can be ignored, it is easy to assess the time ( $\tau$ ) required for creating a spare capacity of the bed due to pressure reduction in it. Setting the velocity of block displacement at 6 cm/yr and the area at 100 km x 100 km gives

$$\tau = \frac{10^3 \text{ km}^3}{10^4 \text{ km}^2 \cdot 6 \cdot 10^{-5} \text{ km/yr}} \approx 1600 \text{ yr}.$$

Fig.6. Scheme of water exchange between the Caspian Sea and the underground reservoir.



No cavity arises above a slowly subsiding block because the water mass in the bed is mobile enough for the newly formed volume to be filled with water almost instantaneously.

For a bed with a permeability of  $10^{-11}$  cm<sup>2</sup>, the flow velocity is defined by the formula:

$$V(\text{cm/s}) = 10^{-2} \frac{\Delta H}{\Delta x}.$$

The pressure gradient  $\Delta H/\Delta x$  is expressed as the difference in water-column height over the length  $\Delta x$ .

The volume released in one year as the basement block subsides is  $\sim 0.6$  km<sup>3</sup>. The water seepage velocity for filling up this volume within one year equals this volume divided by the flow cross-section along the perimeter of the block

$$V = \frac{0,6\text{km}^3/\text{yr}}{4 \cdot 100\text{km} \cdot 0,4\text{km}} \approx 4 \cdot 10^{-3} \text{km}/\text{yr} \approx 10^{-5} \text{cm/s}.$$

This velocity is attained at  $\Delta H/\Delta x = 10^{-3}$ ; i.e., the pressure difference over a length  $\Delta x \approx 100$  km will be only 100 m of water column (10 kg/cm<sup>2</sup>).

The reservoir that arose under the Caspian Sea due to the pore-pressure drop was prerequisite for the seepage flow from the sea into the porous bed to increase

and for conduits to form in fracture zones, through which the sea water was discharged into the underground reservoir in the 1920s-1940s.

When the hydrostatic pressure had been attained, the porous bed underwent a restructuring similar to that occurring in liquefied sand grounds. As water started bearing the load and the porous bed became increasingly compacted, the water pressure grew and by the mid-1970s reached  $1000 \text{ kg/cm}^2$  (the lithostatic pressure is about twice as high as the hydrostatic pressure). The subsidence of the Caspian seabed over ~35 years due to the compaction of the porous layer was viewed as a continuing shoaling of the sea.

As the pore water compressed from  $500 \text{ kg/cm}^2$  to  $1000 \text{ kg/cm}^2$ , the Caspian seabed must have subsided another 1 m (1940-1977). The increasing water pressure changes the direction of seepage flows: water is fed from the underground reservoir into the Caspian. Judging from the rate of sea-level rise, the seawater is supplied to the surface via the same conduits as in the 1920s-1940s.

If this scheme is realistic, it can be predicted that the new stable sea-level highstand will be roughly equal to that in 1920. Another abnormal drop of the Caspian Sea level cannot be expected soon, as the preparatory period related to subsidence of basement blocks lasts ~1000 years.

It might be interesting to estimate the velocity at which a force action is transmitted along a permeable bed. Suppose at a 5-km depth in a bed with a permeability of  $10\text{-}11 \text{ m}^2$ , over a limited area, pressure in the water saturating the bed is equal to the lithostatic pressure. Seepage flow from the high-pressure area will displace the water outside this area, thus increasing in mass. As a result, the high-pressure area will expand and the flow velocity will decrease. Designating the distance covered by the increased-pressure front as  $x$ , at given parameters of the bed the seepage velocity will be  $V = 3\frac{5}{x}$ , where  $V$  is measured in km/yr and  $x$  is measured in km. Taking into account that the increment of water density behind the front on the average equals  $0.0125 \rho$  yields the equation

$$\frac{dx}{dt} = 80V = \frac{1200}{x}$$

Integrating gives  $x \approx 50\sqrt{t}$ . In 25 years, the force-action front will advance 250 km.

## Chapter VI. Natural Bodies

(1) Dynamic structures of solid bodies have a hierarchical organization. They distribute mechanical energy over degrees of freedom, channeling it from larger structural units to ever smaller ones. Such a spreading of energy over structural units of each particular level can be regarded as dissipation of mechanical energy of the body.

En route to the atomic structure, the energy flow passes through a series of variously scaled dissipative levels via branching channels.

The structuring of the solid geospheres can be regarded, at the same time, as a constructive, self-organizing process. From the standpoint of mechanics, all the hierarchical structures are of equal value, as they have the same aim.

On the other hand, it is readily apparent that each of these structures makes its own contribution to the diversity of landscapes.

The functioning of structured bodies in the Earth's interior takes on an intentional purpose only in connection with the reproduction of the natural environment. Geological structures that have a clearly defined designation will here be termed natural bodies. The most important characteristic of natural bodies is that they exchange energy and mass with the environment.

The response of natural bodies to the impact of technology factors can be used to estimate the environmental hazards entailed by various engineering activities.

(2) Let us discuss briefly the essence of some geomechanical problems related to the engineering activity.

The construction of dams and filling of water reservoirs bring about deformation of rock massifs, initiate displacements along flanks of fractures, and change the local seismic regime. A special kind of problems relating to the stability and durability of engineering constructions arises.

It is necessary to assess the impact of the construction on the dynamic structure of the rock massif and on its deformational regime: do natural processes become adapted to the changed landscape, or do the changes in the deformational regime increase with time to eventually result in a restructuring of the solid bodies? Needless to say, only in the former case one can speak about stability and durability of an engineering structure.

Quite a number of new geomechanical problems arise in connection with intense hydrocarbon production in large oil-and-gas provinces. The recovery of large oil and gas volumes and the injection of large water volumes for the reservoir pressure maintenance bear notably on the deformational processes in the upper crustal layers; technologically induced earthquakes are recorded.

Long-range consequences of such interference with natural processes are still unclear.

The slow interactions of an oil-producing formation with the country rocks during hydrocarbon production are not taken into account and remain beyond the scope of any serious study.

At high oil-production rates, half the oil remains in the reservoir unrecovered.

The negative environmental impact of oil and gas production can manifest itself decades later. Meanwhile, if the interaction between the producing formation and the environment is taken into account, oil recovery can be increased.

(3) Controlling the response of a natural body to technological factors can prevent the potentially hazardous upsetting of the existing natural equilibrium.

Of course, every disturbance of the natural equilibrium is not necessarily bound to have negative consequences; environmentally friendly conditions also can arise. Yet predicting a cardinal restructuring of a natural body through disturbance of natural equilibrium even in a limited space is, for the time being, entirely impracticable.

By increasing its power, the industrial civilization gave rise to the environmental crisis. Evidently, it is impossible to remedy the present state of affairs in the physical sphere without using the energy of the natural bodies that participate in the sustenance of the environment. Large-scale construction, mining complexes, and hydrocarbon production have such an impact on the natural structures that they are able to interfere with the distribution of mechanical energy between the natural bodies. To preclude the negative consequences and facilitate the restoration of stable dynamic structures in the environment, it will suffice to harmonize the strategy of development of major industrial objects and production rhythms with the natural bodies functioning in the region.

The control of mechanical energy flows in the Earth's interior eliminates the problem of remote consequences as it was formerly posed.

The reproduction of the environment must become the aim of society's industrial activity in the new century.

## Closing Statement

### 1. On the Steady-state Universe

The homology of structures was ranked by I. Kant among the basic ideas on the structure of the physical world. The striking structural similarity of the atom and the solar system can be to some extent explained by the fact that forces of interaction between bodies change with distance in the same manner in gravity fields and in electrical fields. It is possible, however, that the structural hierarchy is a more general property of the physical world, and the similitude of variously scaled forms is inevitable. Likeness and similitude have been a long-standing basis for the generalization of empirical knowledge.

When extended to the universe, the reasoning applied to the above cases of structural organization of physical bodies makes one give preference to its stationary variant, assuming that the lifetime of the universe is orders of magnitude greater than that of the solar system.

Spatial structures must be reproduced at each hierarchical level with a periodicity proportional to their size.

The steady-state universe, united by the gravity field, must have finite dimensions. Let us assume that one of the conditions constraining its dimensions is the impossibility for light emission to leave its limits. Particles capable of overcoming the gravity of the universe must have energy  $\frac{GMm}{R}$  ( $M$  and  $R$  are the mass and radius of the universe, and  $m$  is the mass of the particle). A quantum of electromagnetic radiation has the energy  $h\nu$  ( $h$  is Planck's constant, and  $\nu$  is the electromagnetic wave frequency), which corresponds to the gravitating mass  $\frac{h\nu}{c^2} = m$ .

Equating these two energies gives

$$\frac{GMm}{R} = mc^2.$$

According to the modern astrophysicists' estimates,  $M \approx 10^{56}$ g. Substituting this value in the formula yields the radius of a universe that does not emit energy beyond its limits ("black hole"):

$$R = \frac{GM}{c^2} \approx 10^{28} \text{ cm.}$$

This value agrees with the commonly accepted one.

All the kinds of mechanical motion within the universe are powered by its gravitational potential. The energy flow from major structures to ever smaller ones provides for their constant reproduction and for the stability of the universe as a whole.

At the lowest atomic level, matter has an enormous energy capacity; here the mechanical energy flow terminates in the chaotic thermal motion.

Accumulated thermal energy is emitted as electromagnetic radiation by celestial bodies into outer space. Traveling away from the center of the universe, the photons lose some energy in the gravity field and, escaping beyond the limits of the physical world, form the photosphere (by analogy with the atmosphere).

The gravitation-compressed plasma inside the stars (compression progresses, as the thermal energy is removed through emission) expends the energy of mechanical motion of particles for nuclear transformations. Nuclear reactions produce the all-penetrating particles (neutrinos), which dissipate energy in outer space.

To sustain the steady-state universe, particles with mass at rest must be reproduced. Whereas the nuclear reactions in the stars' interiors devour the mass at rest, the latter must be generated at the periphery of the universe.

It is very challenging to find the mechanism of reproduction of the mass at rest, but this is the subject of other scientific disciplines.

## ***2. On the Role of the Biosphere in the Earth's Life***

The living matter (a concept introduced by V.N. Vernadsky) of the biosphere is a very complexly structured matter. Innumerable organisms of the plant and animal realms that have attained a high degree of autonomy and socially organized and interacting (although dispersed) insects and bacteria maintain the steady-state process of their reproduction. Living organisms, involving material supplied from the Earth's interior in the cycle of matter, form the atmosphere – the main prerequisite for the existence of the biosphere.

At the expense of the solar radiation energy, complex molecules are produced, which use their chemical potential for subsequent transformations in the living organisms.

Throughout the geological history, the living matter played an active role evidenced by the record it left in the Earth's crust.

The impact of the living matter is strongest on the chemical composition of the atmosphere, on the circulation of the water and carbon dioxide molecules, and on the planet's landscapes and climate.

Mobile structures of the living matter become adapted to the slow changes of natural bodies in the Earth's crust and do not bear directly on the dynamic structures of the Earth's interior. But, influencing the mode of the solar energy consumption and thermodynamic conditions on the Earth's surface, the living matter is capable of changing the rate of erosion and surface mass transport.

The biosphere demonstrates its ability to adjust the reproduction rhythms of the living matter and its structures to the circadian and seasonal insolation changes.

Undoubtedly, the living matter must be sensitive to the electromagnetic radiation in a wide frequency spectrum. Those electromagnetic oscillations important for organic life must include the Earth's natural frequencies. The main tone defined by the travel time of electromagnetic waves around the globe has a frequency of ~10 Hz.

Indirect evidence that living organisms are sensitive to this frequency of oscillations is the fact that human beings react painfully to the 8- to 14-Hz mechanical vibrations of their bodies.

It can be inferred that organisms in the process of structuring of the biosphere may have chosen this frequency to exchange information in the terrestrial environment and with outer space.

It might be interesting to compare the variations of electromagnetic oscillations close to this frequency (multiples of it) with various events in the terrestrial environment and in human society.

### **3. On Natural Philosophy**

The human habitat is a limited region of the physical environment, which is supplemented with the invention of the human genius-the multidimensional and spontaneously developing spiritual sphere.

At the present stage, the empirical information on the environment does not bear directly on human philosophy. This can be interpreted as the tendency of the spiritual sphere to protect itself against the destructive effect of partial, perfunctory information fitted thoughtlessly to everything.

An integral perception of the physical world by means of adjusting empirical data to the ideas and images of the spiritual sphere in a strife for the inner harmony is the purpose and contents of natural philosophy. The physical world is cognizable only within the human habitat where it is represented by natural bodies that have their designation in the reproduction of this habitat. Beyond this environment, the motion of natural bodies is aimless – it is eternal.

The diversity of the sensory perception of the world is reflected in art, which comprehends the laws of harmony and maintains moral norms. Harmony in the spiritual sphere is also the ultimate touchstone of truth in natural science.

An Essay of Geomechanics presents an attempt to interpret the contents of this discipline in terms of natural philosophy.



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